# VAISHALI EDUCATION POINT

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## Differential Equation

**(1.)** A particle moves in a straight line with a velocity given by $\frac{dx}{dt} = x + 1$ (x is the distance described). The time taken by a particle to traverse a distance of 99 metre is

a.) $\log_{10} e$ b.) $2\log_e 10$ c.) $2\log_{10} e$ d.) $\frac{1}{2}\log_{10} e$

**(2.)** Which of the following equation is linear

a.) $\left(\frac{d^2y}{dx^2}\right)^2 + x^2\left(\frac{dy}{dx}\right)^2 = 0$ b.) $y = \frac{dy}{dx} + \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

c.) $\frac{dy}{dx} + \frac{y}{x} = \log x$ d.) $y\frac{dy}{dx} - 4 = x$

**(3.)** The solution of the equation $x^2\frac{d^2y}{dx^2} = \ln x,$ when $x = 1$, $y = 0$ and $\frac{dy}{dx} = -1$ is

a.) $\frac{1}{2}(\ln x)^2 + \ln x$ b.) $\frac{1}{2}(\ln x)^2 - \ln x$ c.) $-\frac{1}{2}(\ln x)^2 + \ln x$ d.) $-\frac{1}{2}(\ln x)^2 - \ln x$

**(4.)** The solution of the equation $\frac{dy}{dx} = \frac{y^2 - y - 2}{x^2 + 2x - 3}$ is

a.) $\frac{1}{3}\log\left|\frac{y-2}{y+1}\right| = \frac{1}{4}\log\left|\frac{x+3}{x-1}\right| + c$ b.) $\frac{1}{3}\log\left|\frac{y+1}{y-2}\right| = \frac{1}{4}\log\left|\frac{x-1}{x+3}\right| + c$

c.) $4\log\left|\frac{y-2}{y+1}\right| = 3\log\left|\frac{x-1}{x+3}\right| + c$ d.) None of these

**(5.)** Degree of the given differential equation $\left(\frac{d^2y}{dx^2}\right)^3 = \left(1 + \frac{dy}{dx}\right)^{1/2}$, is

a.) 2 b.) 3 c.) $\frac{1}{2}$ d.) 6

**(6.)** The differential equation whose solution is $y = A\sin x + B\cos x,$ is

a.) $\frac{d^2y}{dx^2} + y = 0$ b.) $\frac{d^2y}{dx^2} - y = 0$ c.) $\frac{dy}{dx} + y = 0$ d.) None of these

**(7.)** Solution of differential equation $\frac{dy}{dx}=\frac{y-x}{y+x}$ is

a.) $\log_e(x^2+y^2)+2\tan^{-1}\frac{y}{x}+c=0$

b.) $\frac{y^2}{2}+xy=xy-\frac{x^2}{2}+c$

c.) $\left(1+\frac{x}{y}\right)y=\left(1-\frac{x}{y}\right)x+c$

d.) $y=x-2\log_e y+c$

**(8.)** The solution of $ye^{-x/y}dx-(xe^{-x/y}+y^3)dy=0$ is

a.) $\frac{y^2}{2}+e^{-x/y}=k$ b.) $\frac{x^2}{2}+e^{-x/y}=k$ c.) $\frac{x^2}{2}+e^{x/y}=k$ d.) $\frac{y^2}{2}+e^{x/y}=k$

**(9.)** The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{\frac{2}{3}}=4\frac{d^3y}{dx^3}$ are

a.) $1,\frac{2}{3}$ b.) 3, 1 c.) 3, 3 d.) 1, 2

**(10.)** The solution of the equation $(x+2y^3)\frac{dy}{dx}-y=0$ is Where A is any arbitrary constant

a.) $y(1-xy)=Ax$ b.) $y^3-x=Ay$ c.) $x(1-xy)=Ay$ d.) $x(1+xy)=Ay$

**(11.)** The equation of the curve through the point (1,0) and whose slope is $\frac{y-1}{x^2+x}$ is

a.) $(y-1)(x+1)+2x=0$

b.) $2x(y-1)+x+1=0$

c.) $x(y-1)(x+1)+2=0$

d.) None of these

**(12.)** Solution of differential equation $2xy\frac{dy}{dx}=x^2+3y^2$ is (where p is a constant)

a.) $x^3+y^2=px^2$

b.) $\frac{x^2}{2}+\frac{y^3}{x}=y^2+p$

c.) $x^2+y^3=px^2$

d.) $x^2+y^2=px^3$

**(13.)** If c is any arbitrary constant, then the general solution of the differential equation $ydx-xdy=xy\,dx$ is given by

a.) $y=cx\,e^{-x}$ b.) $x=cye^{-x}$ c.) $y+e^x=cx$ d.) $ye^x=cx$

**(14.)** The differential equation for all the straight lines which are at a unit distance from the origin is

a.) $\left(y-x\frac{dy}{dx}\right)^2=1-\left(\frac{dy}{dx}\right)^2$

b.) $\left(y+x\frac{dy}{dx}\right)^2=1+\left(\frac{dy}{dx}\right)^2$

c.) $\left(y-x\frac{dy}{dx}\right)^2=1+\left(\frac{dy}{dx}\right)^2$

d.) $\left(y+x\frac{dy}{dx}\right)^2=1-\left(\frac{dy}{dx}\right)^2$

**(15.)** The general solution of the differential equation $ydx+(1+x^2)\tan^{-1}xdy=0,$ is

a.) $y\tan^{-1}x=c$ b.) $x\tan^{-1}y=c$ c.) $y+\tan^{-1}x=c$ d.) $x+\tan^{-1}y=c$

**(16.)** The degree of the differential equation $\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/4} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$ is

a.) $\frac{1}{3}$ b.) 4 c.) 9 d.) $\frac{3}{4}$

**(17.)** The differential equation of all circles which passes through the origin and whose centre lies on y-axis, is

a.) $(x^2 - y^2)\frac{dy}{dx} - 2xy = 0$ b.) $(x^2 - y^2)\frac{dy}{dx} + 2xy = 0$

c.) $(x^2 - y^2)\frac{dy}{dx} - xy = 0$ d.) $(x^2 - y^2)\frac{dy}{dx} + xy = 0$

**(18.)** If $\frac{dy}{dx} = \frac{xy + y}{xy + x}$, then the solution of the differential equation is

a.) $y = xe^x + c$ b.) $y = e^x + c$ c.) $y = Axe^{x-y}$ d.) $y = x + A$

**(19.)** The degree and order of the differential equation of the family of all parabolas whose axis is x–axis, are respectively

a.) 2, 1 b.) 1, 2 c.) 3, 2 d.) 2, 3

**(20.)** Integrating factor of equation $(x^2 + 1)\frac{dy}{dx} + 2xy = x^2 - 1$ is

a.) $x^2 + 1$ b.) $\frac{2x}{x^2 + 1}$ c.) $\frac{x^2 - 1}{x^2 + 1}$ d.) None of these

**(21.)** A function $y = f(x)$ has a second order derivatives $f''(x) = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is

a.) $(x+1)^3$ b.) $(x-1)^3$ c.) $(x+1)^2$ d.) $(x-1)^2$

**(22.)** $(x^2 + y^2)dy = xydx$ . If $y(x_0) = e$ , $y(1) = 1$ , then value of $x_0 =$

a.) $\sqrt{3}e$ b.) $\sqrt{e^2 - \frac{1}{2}}$ c.) $\sqrt{\frac{e^2 - 1}{2}}$ d.) $\sqrt{\frac{e^2 + 1}{2}}$

**(23.)** The order of the differential equation whose general solution is given by $y = C_1 e^{2x + C_2} + C_3 e^x + C_4 \sin(x + C_5)$ is

a.) 5 b.) 4 c.) 3 d.) 2

**(24.)** Solution of the differential equation, $y\,dx - x\,dy + xy^2 dx = 0$ can be

a.) $2x + x^2 y = $ ª$y$ b.) $2y + y^2 x = $ ª$y$ c.) $2y - y^2 x = $ ª$y$ d.) None of these

**(25.)** Equation of curve passing through (3, 9) which satisfies the differential equation $\frac{dy}{dx} = x + \frac{1}{x^2}$, is

a.) $6xy = 3x^2 - 6x + 29$ b.) $6xy = 3x^3 - 29x + 6$

c.) $6xy = 3x^3 + 29x - 6$ d.) None of these

**(26.)** The equation of the curve passing through the origin and satisfying the equation $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$ is

a.) $3(1+x^2)y = 4x^3$ b.) $3(1-x^2)y = 4x^3$ c.) $3(1+x^2) = x^3$ d.) None of these

**(27.)** The differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right)^{1/2} = y^3$ has the degree

a.) 1/2 b.) 2 c.) 3 d.) 4

**(28.)** The differential equation of all circles in the first quadrant which touch the coordinate axes is of order

a.) 1 b.) 2 c.) 3 d.) None of these

**(29.)** For solving $\frac{dy}{dx} = (4x + y + 1)$, suitable substitution is

a.) $y = vx$ b.) $y = 4x + v$ c.) $y = 4x$ d.) $y + 4x + 1 = v$

**(30.)** The differential equation of all the lines in the xy-plane is

a.) $\frac{dy}{dx} - x = 0$ b.) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$ c.) $\frac{d^2y}{dx^2} = 0$ d.) $\frac{d^2y}{dx^2} + x = 0$

**(31.)** If the gradient of the tangent at any point (x, y) of a curve which passes through the point $\left(1, \frac{®}{4}\right)$ is $\left\{\frac{y}{x} - \sin^2\left(\frac{y}{x}\right)\right\}$, then equation of the curve is

a.) $y = \cot^{-1}(\log_e x)$ b.) $y = \cot^{-1}\left(\log_e \frac{x}{e}\right)$

c.) $y = x\cot^{-1}(\log_e ex)$ d.) $y = \cot^{-1}\left(\log_e \frac{e}{x}\right)$

**(32.)** Integrating factor of differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$ is

a.) $\cos x$ b.) $\tan x$ c.) $\sec x$ d.) $\sin x$

**(33.)** The slope of the tangent at (x, y) to a curve passing through a point (2, 1) is $\frac{x^2 + y^2}{2xy}$, then the equation of the curve is

a.) $2(x^2 - y^2) = 3x$ b.) $2(x^2 - y^2) = 6y$ c.) $x(x^2 - y^2) = 6$ d.) $x(x^2 + y^2) = 10$

**(34.)** The degree of the differential equation $\frac{d^2y}{dx^2} - \sqrt{\frac{dy}{dx} - 3} = x$ is

a.) 2 b.) 1 c.) 1/2 d.) 3

**(35.)** If $x^2 + y^2 = 1$ then $\left(y' = \frac{dy}{dx}, y'' = \frac{d^2y}{dx^2}\right)$

a.) $yy'' - 2(y')^2 + 1 = 0$ b.) $yy'' + (y')^2 + 1 = 0$

c.) $yy'' - (y')^2 - 1 = 0$ d.) $yy'' + 2(y')^2 + 1 = 0$

**(36.)** The differential equation $\cot y \; dx = x \; dy$ has a solution of the form

a.) $y = \cos x$ b.) $x = c\sec y$ c.) $x = \sin y$ d.) $y = \sin x$

**(37.)** The order and the degree of differential equation $\frac{d^4y}{dx^4} - 4\frac{d^3y}{dx^3} + 8\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 4y = 0$ are respectively

a.) 4, 1 b.) 1, 4 c.) 1, 1 d.) None of these

**(38.)** The differential equation for which $\sin^{-1} x + \sin^{-1} y = c$ is given by

a.) $\sqrt{1-x^2}\; dx + \sqrt{1-y^2}\; dy = 0$ b.) $\sqrt{1-x^2}\; dy + \sqrt{1-y^2}\; dx = 0$

c.) $\sqrt{1-x^2}\; dy - \sqrt{1-y^2}\; dx = 0$ d.) $\sqrt{1-x^2}\; dx - \sqrt{1-y^2}\; dy = 0$

**(39.)** If $\frac{dy}{dx} = e^{-2y}$ and $y = 0$ when $x = 5,$ the value of x for $y = 3$ is

a.) $e^5$ b.) $e^6 + 1$ c.) $\frac{e^6 + 9}{2}$ d.) $\log_e 6$

**(40.)** The differential equation of the family of parabolas with focus at the origin and the x–axis as axis is

a.) $y\left(\frac{dy}{dx}\right)^2 + 4x\frac{dy}{dx} = 4y$ b.) $-y\left(\frac{dy}{dx}\right)^2 = 2x\frac{dy}{dx} - y$

c.) $y\left(\frac{dy}{dx}\right)^2 + y = 2xy\frac{dy}{dx}$ d.) $y\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} + y = 0$

**(41.)** The slope of the tangent at $(x, y)$ to a curve passing through $\left(1, \frac{®}{4}\right)$ is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$, then the equation of the curve is

a.) $y = \tan^{-1}\left[\log\left(\frac{e}{x}\right)\right]$ b.) $y = x\tan^{-1}\left[\log\left(\frac{x}{e}\right)\right]$

c.) $y = x\tan^{-1}\left[\log\left(\frac{e}{x}\right)\right]$ d.) None of these

**(42.)** The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^{\frac{1}{4}} = 0$ are respectively a.) 2, 3 b.) 3, 3 c.) 2, 6 d.) 2, 4

**(43.)** The solution of $y' - y = 1, y(0) = -1$ is given by $y(x) =$

a.) $-\exp(x)$ b.) $-\exp(-x)$ c.) $-1$ d.) $\exp(x) - 2$

**(44.)** $y = \frac{x}{x+1}$ is a solution of the differential equation

a.) $y^2 \frac{dy}{dx} = x^2$ b.) $x^2 \frac{dy}{dx} = y^2$ c.) $y \frac{dy}{dx} = x$ d.) $x \frac{dy}{dx} = y$

**(45.)** The equation of family of curves for which the length of the normal is equal to the radius vector is

a.) $y^2 \pm x^2 = k$ b.) $y \pm x = k$ c.) $y^2 = kx$ d.) None of these

**(46.)** The degree of the differential equation $y(x) = 1 + \frac{dy}{dx} + \frac{1}{1.2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{1.2.3}\left(\frac{dy}{dx}\right)^3 + \ldots$ is

a.) 2 b.) 3 c.) 1 d.) None of these

**(47.)** The differential equation of all straight lines passing through the origin is

a.) $y = \sqrt{x \frac{dy}{dx}}$ b.) $\frac{dy}{dx} = y + x$ c.) $\frac{dy}{dx} = \frac{y}{x}$ d.) None of these

**(48.)** If $\frac{dy}{dx} = 1 + x + y + xy$ and $y(-1) = 0$, then function y is

a.) $e^{(1-x)^2/2}$ b.) $e^{(1+x)^2/2} - 1$ c.) $\log_e(1+x) - 1$ d.) $1 + x$

**(49.)** The differential equation of the family of curves $v = \frac{A}{r} + B$, where A and B are arbitrary constants, is

a.) $\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} = 0$ b.) $\frac{d^2v}{dr^2} - \frac{2}{r}\frac{dv}{dr} = 0$ c.) $\frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr} = 0$ d.) None of these

**(50.)** Family $y = Ax + A^3$ of curve represented by the differential equation of degree

a.) Three b.) Two c.) One d.) None of these

# ANSWER

| Differential Equation | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **(1.)** | b | **(2.)** | c | **(3.)** | d | **(4.)** | c | **(5.)** | d |
| **(6.)** | a | **(7.)** | a | **(8.)** | a | **(9.)** | c | **(10.)** | b |
| **(11.)** | a | **(12.)** | d | **(13.)** | d | **(14.)** | c | **(15.)** | a |
| **(16.)** | b | **(17.)** | a | **(18.)** | c | **(19.)** | b | **(20.)** | a |
| **(21.)** | b | **(22.)** | a | **(23.)** | b | **(24.)** | a | **(25.)** | c |
| **(26.)** | a | **(27.)** | d | **(28.)** | a | **(29.)** | d | **(30.)** | c |
| **(31.)** | c | **(32.)** | c | **(33.)** | a | **(34.)** | a | **(35.)** | b |
| **(36.)** | b | **(37.)** | a | **(38.)** | b | **(39.)** | c | **(40.)** | b |
| **(41.)** | c | **(42.)** | a | **(43.)** | c | **(44.)** | b | **(45.)** | a |
| **(46.)** | c | **(47.)** | c | **(48.)** | b | **(49.)** | c | **(50.)** | a |

# EXPLANATION

## Differential Equation

**(1.)** $\frac{dx}{dt} = x + 1$

$\Rightarrow \log(x+1) = t + c$

Putting $t = 0, x = 0$,

we get $\log 1 = c$

$\Rightarrow c = 0$

$\therefore t = \log(x+1)$.

For $x = 99, t = \log_e 100 = 2\log_e 10$.

**(2.)** $\frac{dy}{dx} + \frac{y}{x} = \log x$

is a linear differential equation.

**(3.)** $\frac{d^2y}{dx^2} = \frac{\log x}{x^2} \Rightarrow \frac{dy}{dx} = \frac{-(\log x + 1)}{x} + c$

At $\frac{dy}{dx} = -1$, $x = 1, y = 0, \therefore c = 0$

$\Rightarrow y = -\int \frac{\log x + 1}{x} dx = -\frac{1}{2}(\log x)^2 - \log x$.

**(4.)** $\frac{dy}{dx} = \frac{y^2 - y - 2}{x^2 + 2x - 3}$

$\Rightarrow \frac{dy}{(y-2)(y+1)} = \frac{dx}{(x+3)(x-1)}$

$\Rightarrow \int \frac{dy}{(y-2)(y+1)} = \int \frac{dx}{(x+3)(x-1)}$

$\Rightarrow \frac{1}{3}\int \left(\frac{1}{y-2} - \frac{1}{y+1}\right)dy = \frac{1}{4}\int \left(\frac{1}{x-1} - \frac{1}{x+3}\right)dx$

$\Rightarrow \frac{1}{3}\log\left|\frac{y-2}{y+1}\right| = \frac{1}{4}\left|\frac{x-1}{x+3}\right| + c$.

**(5.)** On squaring, we will have the result.

**(6.)** $y = A\sin x + B\cos x$

$\Rightarrow \frac{dy}{dx} = A\cos x - B\sin x$

$\Rightarrow \frac{d^2y}{dx^2} = -A\sin x - B\cos x$

$= -(A\sin x + B\cos x) = -y$

$\Rightarrow \frac{d^2y}{dx^2} + y = 0$

is the required differential equation.

**(7.)** Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore v + x\frac{dv}{dx} = \frac{v-1}{v+1}$

$\Rightarrow x\frac{dv}{dx} = \frac{v-1}{v+1} - v$

$\Rightarrow x\dfrac{dv}{dx} = -\dfrac{v^2+1}{v+1}$

$\Rightarrow \int \dfrac{dx}{x} = -\int \dfrac{v+1}{v^2+1}\,dv$

$\Rightarrow -\log_e x = \dfrac{1}{2}\int \dfrac{2v}{v^2+1}dv + \int \dfrac{1}{v^2+1}dv$

$\Rightarrow -\log_e x = \dfrac{1}{2}\log(v^2+1) + \tan^{-1} v + c$

$\Rightarrow -2\log_e x = \log\left(\dfrac{x^2+y^2}{x^2}\right) + 2\tan^{-1}\left(\dfrac{y}{x}\right) + c$

$\Rightarrow \log_e(x^2+y^2) + 2\tan^{-1}\dfrac{y}{x} + c = 0$ .

**(8.)** $y\,e^{-x/y}dx - (xe^{-x/y} + y^3)dy = 0$

$e^{-x/y}(ydx - xdy) = y^3 dy$

$\Rightarrow e^{-x/y}\dfrac{(ydx - xdy)}{y^2} = ydy$

$e^{-x/y}d\left(\dfrac{x}{y}\right) = ydy$ . Integrating both sides, we get

$k - e^{-x/y} = \dfrac{y^2}{2} \Rightarrow \dfrac{y^2}{2} + e^{-x/y} = k$

**(9.)** To check, order and degree, the given differential equation should be free from radicals, hence taking cube on both sides, $\left(1 + 3.\dfrac{dy}{dx}\right)^2 = \left(4.\dfrac{d^3y}{dx^3}\right)^3$

Clearly, order = 3, degree = 3.

**(10.)** $(x+2y^3)\dfrac{dy}{dx} = y \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x+2y^3}$

$\Rightarrow \dfrac{dx}{dy} = \dfrac{x+2y^3}{y}$ or $\dfrac{dx}{dy} - \dfrac{x}{y} = 2y^2$ , which is a linear equation of the form $\dfrac{dx}{dy} + Px = Q$

So, integrating factor (I.F.) $= e^{-\int \frac{1}{y}dy}$ and solution is $x\dfrac{1}{y} = \int \dfrac{1}{y}2y^2dy + A = y^2 + A$

$\Rightarrow x = y^3 + Ay$

$\Rightarrow y^3 - x = Ay$; where A can be $-ve$ or $+ve$ .

**(11.)** Slope $= \dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y-1}{x^2+x}$

$\Rightarrow \dfrac{dy}{y-1} = \dfrac{dx}{x^2+x}$

$\Rightarrow \int \dfrac{1}{y-1}dy = \int \left(\dfrac{1}{x} - \dfrac{1}{x+1}\right)dx + c$

$\Rightarrow \dfrac{(y-1)(x+1)}{x} = k$

Putting $x = 1$ , $y = 0$ ,

we get $k = -2$

Hence the equation is

$(y-1)(x+1) + 2x = 0$ .

**(12.)** It is homogeneous equation $\dfrac{dy}{dx} = \dfrac{x^2+3y^2}{2xy}$

Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So, we get $x\frac{dv}{dx} = \frac{1+v^2}{2v}$

$\Rightarrow \frac{2vdv}{1+v^2} = \frac{dx}{x}$

On integrating, we get $x^2 + y^2 = px^3$ .

**(13.)** Given $ydx - xdy = xydx$

$\Rightarrow \frac{ydx - xdy}{xy} = dx$

$\Rightarrow d\left[\ln\left(\frac{x}{y}\right)\right] = dx$

Integrating both sides,

we get $\ln\frac{x}{y} + \ln c = x$

$\Rightarrow ye^x = cx$ .

**(14.)** Since the equation of lines whose distance from origin is unit, is given by

$x\cos\sim + y\sin\sim = 1$ .....(i)

Differentiate w.r.t. x, we get

$\cos\sim + \frac{dy}{dx}\sin\sim = 0$ .....(ii)

On eliminating the $'\sim'$ with the help of (i) and (ii)

i.e., (i) –x × (ii)

$\Rightarrow \sin\sim\left(y - x\frac{dy}{dx}\right) = 1$

$\Rightarrow \left(y - x\frac{dy}{dx}\right) = \operatorname{cosec}\sim$ .....(iii)

Also (ii) $\Rightarrow \frac{dy}{dx} = -\cot\sim$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \cot^2\sim$ .....(iv)

Therefore by (iii) and (iv),

$1 + \left(\frac{dy}{dx}\right)^2 = \left(y - x\frac{dy}{dx}\right)^2$ .

**(15.)** $ydx + (1+x^2)\tan^{-1}xdy = 0$

$\Rightarrow \int\frac{dx}{(1+x^2)\tan^{-1}x} = -\int\frac{dy}{y}$

$\Rightarrow \log(\tan^{-1}x) + \log c = -\log y$

$\Rightarrow \log(y\tan^{-1}x) + \log c = 0$

$\Rightarrow y\tan^{-1}x = c$ .

**(16.)** $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/4} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$

$\Rightarrow \left[1 + \left(\frac{dy}{dx}\right)^2\right]^9 = \left(\frac{d^2y}{dx^2}\right)^4$

Clearly, degree is 4.

**(17.)** The system of circles pass through origin and centre lies on y-axis is $x^2 + y^2 - 2ay = 0$

$\Rightarrow\ 2x + 2y\dfrac{dy}{dx} - 2a\dfrac{dy}{dx} = 0$

$\Rightarrow\ 2a = 2y + 2x\dfrac{dx}{dy}$

Therefore, the required differential equation is

$x^2 + y^2 - 2y^2 - 2xy\dfrac{dx}{dy} = 0$

$\Rightarrow (x^2 - y^2)\dfrac{dy}{dx} - 2xy = 0$ .

**(18.)** $\dfrac{dy}{dx} = \dfrac{xy + y}{xy + x}$

$\Rightarrow\ \left(\dfrac{1+y}{y}\right)dy = \left(\dfrac{1+x}{x}\right)dx$

On integrating both sides, we get

$\log y + y = \log x + x + \log A$

$\Rightarrow\ \log\left(\dfrac{y}{Ax}\right) = x - y$

$\Rightarrow y = Axe^{x-y}$ .

**(19.)** $y^2 = \pm 4a(x-h)$

$\Rightarrow\ 2yy_1 = \pm 4a$

$\Rightarrow\ yy_1 = \pm 2a$

$\Rightarrow\ y_1^2 + yy_2 = 0$

Hence degree = 1, order = 2.

**(20.)** $\dfrac{dy}{dx} + \dfrac{2x}{1+x^2}y = \dfrac{x^2-1}{x^2+1}$

I.F. $= e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1 + x^2$ .

**(21.)** Given $f''(x) = 6(x-1)$

$f'(x) = 3(x-1)^2 + c_1$ …..(i)

But at point (2, 1) the line $y = 3x - 5$ is tangent to the graph $y = f(x)$ . Hence $\left.\dfrac{dy}{dx}\right|_{x=2} = 3$ or $f'(2) = 3$ .

Then from (i) $f'(2) = 3(2-1)^2 + c_1$

$3 = 3 + c_1 \Rightarrow c_1 = 0$ i.e., $f'(x) = 3(x-1)^2$

Given $f(2) = 1$

$f(x) = (x-1)^3 + c_2$

$\Rightarrow\ f(2) = 1 + c_2$

$\Rightarrow\ 1 = 1 + c_2$

$\Rightarrow\ c_2 = 0$

Hence $f(x) = (x-1)^3$ .

**(22.)** $x^2dy + y^2dy = xydx$

$\Rightarrow\ x(xdy - ydx) = -y^2dy$

$\Rightarrow\ x\dfrac{(ydx - xdy)}{y^2} = dy$

$\Rightarrow\ \dfrac{x}{y}d\left(\dfrac{x}{y}\right) = \dfrac{dy}{y}$

Integrating, $\frac{x^2}{2y^2} = \log_e y + c$

Given $y(1) = 1$

$\Rightarrow c = \frac{1}{2}$

$\Rightarrow \frac{x^2}{2y^2} = \log_e y + \frac{1}{2}$

Now $y(x_0) = e$

$\Rightarrow \frac{x_0^2}{2e^2} - \log_e e - \frac{1}{2} = 0$

$\Rightarrow x_0^2 = 3e^2$

$\Rightarrow x_0 = \pm\sqrt{3}e$

**(23.)** $y = C_1 e^{2x+C_2} + C_3 e^x + C_4 \sin(x + C_5)$

$= C_1 . e^{C_2} e^{2x} + C_3 e^x + C_4(\sin x \cos C_5 + \cos x \sin C_5)$

$= Ae^{2x} + C_3 e^x + B \sin x + D \cos x$

Here, $A = C_1 e^{C_2}$ , $B = C_4 \cos C_5$ , $D = C_4 \sin C_5$

(Since equation consists of four arbitrary constants)

$\therefore$ order of differential equation = 4.

**(24.)** $\frac{ydx - xdy}{y^2} = -xdx$

$\Rightarrow d\left(\frac{x}{y}\right) = -xdx$

Integrating both side, we get $\frac{x}{y} = -\frac{x^2}{2} + c$

$\Rightarrow 2x + x^2 y = 2cy$

$\Rightarrow 2x + x^2 y =$ ª$y$ [ ª $= 2c$ ]

**(25.)** $\frac{dy}{dx} = x + \frac{1}{x^2}$

$\Rightarrow \int dy = \int \left(x + \frac{1}{x^2}\right) dx$

$\Rightarrow y = \frac{x^2}{2} - \frac{1}{x} + c$

Since it passes through (3, 9), therefore

$9 = \frac{9}{2} - \frac{1}{3} + c$

$\Rightarrow c = \frac{29}{6}$

$\therefore y = \frac{x^2}{2} - \frac{1}{x} + \frac{29}{6}$

$\Rightarrow 6xy = 3x^3 + 29x - 6$ .

**(26.)** $\frac{dy}{dx} + \frac{2x}{1 + x^2} y = \frac{4x^2}{1 + x^2}$

It is linear equation of the form $\frac{dy}{dx} + Py = Q$

Here $P = \frac{2x}{1 + x^2}$ and $Q = \frac{4x^2}{1 + x^2}$

I.F. $= e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = (1 + x^2)$

Therefore, solution is given by

$$y.(1+x^2)=\int \frac{4x^2}{1+x^2}(1+x^2)dx+c=\frac{4x^3}{3}+c.$$

But it passes through (0,0) therefore $c=0$, hence the curve is $3y(1+x^2)=4x^3$.

**(27.)** From the given equation,

$$\left(\left(\frac{d^2y}{dx^2}\right)^2-y^3\right)^2=\frac{dy}{dx}.$$

Hence, it is obvious from the equation that degree is 4.

**(28.)** The equation of the family of circle which touch both the axes is $(x-a)^2+(y-a)^2=a^2$, where a is a parameter. This is one parameter family of curves. So its differential equation is of order one.

**(29.)** Put $y+4x+1=v$.

**(30.)** The equation of all the lines in xy-plane is given by $y=mx+c$

Differentiate it twice w.r.t. x, we get $\frac{d^2y}{dx^2}=0$.

**(31.)** Given $\frac{dy}{dx}=\frac{y}{x}-\sin^2\left(\frac{y}{x}\right)$

Put y = vx

$$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=v-\sin^2 v$$

$$\Rightarrow -\operatorname{cosec}^2 v\,dv=\frac{dx}{x}$$

Integrating both sides, $-\int \operatorname{cosec}^2 v\,dv=\int\frac{dx}{x}$

$$\Rightarrow \cot v=\log x+c,\ \cot\frac{y}{x}=\log x+c$$

This curve passes through the point $\left(1,\frac{®}{4}\right)$

$\therefore$ c = 1

$$\Rightarrow \cot\frac{y}{x}=\log x+\log_e e$$

$$\cot\frac{y}{x}=\log x\,e$$

$$\Rightarrow y=x\cot^{-1}(\log x\,e).$$

**(32.)** $\frac{dy}{dx}+y\tan x=\sec x$

I.F. $=e^{\int \tan x\,dx}=e^{\log\sec x}=\sec x$

**(33.)** $\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$.

Put $y=vx$

$$\Rightarrow v+x.\frac{dv}{dx}=\frac{x^2+v^2x^2}{2vx^2}$$

$$\frac{2v}{1-v^2}.dv=\frac{dx}{x}$$

Integrating both sides,

$-\log(1-v^2)=\log x+\log c$

$-\log\left(1-\dfrac{y^2}{x^2}\right)=\log x+\log c$ …..(i)

This passes through $(2,1)$

$-\log\left(1-\dfrac{1}{4}\right)=\log 2+\log c$

$\Rightarrow\ c=\dfrac{2}{3}$

From equation (i), $\log\left(\dfrac{x^2}{x^2-y^2}\right)=\log xc$

$2(x^2-y^2)=3x$ .

**(34.)** $\dfrac{d^2y}{dx^2}-\sqrt{\dfrac{dy}{dx}-3}=x\Rightarrow\dfrac{d^2y}{dx^2}-x=\sqrt{\dfrac{dy}{dx}-3}$

Squaring both sides, we get

$\left(\dfrac{d^2y}{dx^2}-x\right)^2=\left(\dfrac{dy}{dx}-3\right)$

$\Rightarrow\left(\dfrac{d^2y}{dx^2}\right)^2+x^2-2x\dfrac{d^2y}{dx^2}=\dfrac{dy}{dx}-3$ .

Clearly, degree = 2.

**(35.)** Differentiating w.r.t. $x$, $2x+2yy'=0$

or $x+yy'=0$

Differentiating again w.r.t. $x$, $1+y'^2+yy''=0$ .

**(36.)** $\cot y.dx=x.dy$

$\Rightarrow\ \dfrac{dx}{x}=\dfrac{dy}{\cot y}$

$\Rightarrow\ \dfrac{dx}{x}=\tan y.dy$

Integrating both sides,

$\log x=\log\sec y+\log c$

$\Rightarrow\ x=c\sec y$ .

**(37.)** Order is 4 and degree is 1.

**(38.)** Given equation is

$\sin^{-1}x+\sin^{-1}y=c$ .....(i)

On differentiating w.r.t. to x, we get

$\dfrac{1}{\sqrt{1-x^2}}+\dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx}=0$

$\Rightarrow\dfrac{dy}{dx}=-\dfrac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\Rightarrow\ \sqrt{1-x^2}\ dy+\sqrt{1-y^2}\ dx=0$ .

**(39.)** $\int\dfrac{dy}{e^{-2y}}=\int dx$

$\Rightarrow\ \dfrac{e^{2y}}{2}=x+c$

At $x=5, y=0$

$\Rightarrow\ c = -\frac{9}{2}$;

$\therefore$ For $y = 3,\ x = \frac{e^6 + 9}{2}$.

**(40.)** Equation of family of parabolas with focus at $(0,0)$ and x-axis as axis is

$y^2 = 4a(x + a)$ …..(i)

Differentiating (i) with respect to x,

$2yy_1 = 4a;\ y^2 = 2yy_1\left(x + \frac{yy_1}{2}\right)$

$y = 2xy_1 + yy_1^2$

$\Rightarrow\ y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} = y$.

**(41.)** We have $\frac{dy}{dx} = \frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$

Putting $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$v + x\frac{dv}{dx} = v - \cos^2 v$ or $\frac{dv}{\cos^2 v} = -\frac{dx}{x}$

On integrating, we get $\tan v = -\log x + \log c$

$\Rightarrow\ \tan\left(\frac{y}{x}\right) = -\log x + \log C$

This passes through $\left(1, \frac{®}{4}\right)$, therefore $1 = \log c$

$\Rightarrow \tan\left(\frac{y}{x}\right) = -\log x + \log e$

$\Rightarrow y = x\tan^{-1}\left[\log\left(\frac{e}{x}\right)\right]$.

**(42.)** Given $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$

Taking cube, $\left[\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4}\right]^3 = 0$

Order of highest derivative $= 2$

Degree of highest derivative $= 3$ .

**(43.)** $\frac{dy}{dx} - y = 1\ \Rightarrow\ \frac{dy}{1+y} = dx$

Integrating both sides

$\log(1 + y) = x + c$

$\Rightarrow\ 1 + y = e^x.e^c$

$\because x = 0,\ y = -1$.

Then, $1 - 1 = e.e^c$

$\Rightarrow e^c = 0$

Therefore solution

$1 + y = e^x \times 0 \Rightarrow y(x) = -1$.

**(44.)** $y = \frac{x}{x+1} \Rightarrow \frac{1}{y} = 1 + \frac{1}{x}\ -\frac{1}{y^2}\frac{dy}{dx} = 0 - \frac{1}{x^2}$

$\Rightarrow\ x^2\frac{dy}{dx} = y^2$.

**(45.)** Length of the normal $= y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

It is given that $y\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{x^2+y^2}$

($\because$ Radius vector $= r = \sqrt{x^2+y^2}$ )

$\Rightarrow y^2 + y^2\left(\frac{dy}{dx}\right)^2 = x^2 + y^2$

$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2 = x^2$

$\Rightarrow ydy \pm xdx = 0$

$\Rightarrow y^2 \pm x^2 = k$ .

**(46.)** $y = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + ... + \infty$ where $t = \frac{dy}{dx}$

$\Rightarrow y = e^t$ , $\therefore\ t = \log y$

$\Rightarrow \frac{dy}{dx} = \log y$ .

Hence degree is 1.

**(47.)** The equation of all straight lines passing through the origin is

$y = mx$ …..(i)

where m is arbitrary constant

Differentiate (i) w.r.t. x, we get

$\frac{dy}{dx} = m \Rightarrow \frac{dy}{dx} = \frac{y}{x}$, (By (i)).

**(48.)** $\frac{dy}{dx} = 1 + x + y + xy$

$\Rightarrow \frac{dy}{dx} = (1+x) + y(1+x)$

$\Rightarrow \frac{dy}{dx} = (1+x)(1+y)$

$\Rightarrow \frac{dy}{(1+y)} = dx(1+x)$

Integrating both sides, $\int \frac{dy}{(1+y)} = \int dx(1+x)$

$\log(1+y) = x + \frac{x^2}{2} + \log c$

$y = ce^{x+(x^2/2)} - 1$

$\Rightarrow y(-1) = ce^{-1+(1/2)} - 1 = 0$

$\therefore ce^{-1/2} = 1 \Rightarrow c = e^{1/2}$

$\therefore y = e^{1/2}e^{x+\frac{x^2}{2}} - 1$ , $y = e^{\frac{(x+1)^2}{2}} - 1$ .

**(49.)** $\frac{dv}{dr} = -\frac{A}{r^2} + 0$

$\Rightarrow \frac{d^2v}{dr^2} = \frac{2A}{r^3}$

$\Rightarrow \frac{d^2v}{dr^2} = \frac{2}{r}\left(\frac{A}{r^2}\right)$

$\Rightarrow \frac{d^2v}{dr^2} = \frac{2}{r}\left(-\frac{dv}{dr}\right)$

$\Rightarrow \frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr} = 0$ .

**(50.)** Differentiating the given equation,

we get $\frac{dy}{dx} = A$

$\therefore y = x\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^3$ which is of degree 3.