VAISHALI EDUCATION POINT

(Quality Education Provider)

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SAMPLE PAPER – 2009
Class – XII
Subject – Mathematics

General Instructions:

1 All questions are compulsory
2 Q 1 – 10 carries 1 marks, Q 11 – 22 carries 4 marks Q-23to 29 carries 6 marks

SECTION – A

1. Construct a 2×2 matrix whose elements are given by $a_{ij}=|2i-j|$.

2. If A is a square matrix of order 3 such that $|adj\,A|=144$, write the value of $|A|$.

3. If $0<x<\pi$ and the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular, write the value(s) of x.

4. If $f:R\to R$ is defined by $f(x)=x^2-2x+3$, write the value of $f(f(x))$.

5. Write the value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{5\pi}{6}\right)$.

6. Write the value of $\int\frac{dx}{\sin^2 x\cos^2 x}$.

7.Write a differential equation representing family of the curves $y=A\cos(x+B)$, where A and B are parameters.

8. Write the position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ externally in the ratio 2:1 , where $\vec{a}=\hat{i}+2\hat{j}-k$ and $\vec{b}=-\hat{i}+\hat{j}+k$.

9. Write the value of $|\vec{a}-\vec{b}|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=2$, $|\vec{b}|=3$ and $\vec{a}.\vec{b}=4$.

10. If a line makes α,β,γ with the x-axis, y-axis and z-axis respectively, then write the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$.

SECTION – B

11. Consider $f:\mathrm{R}_+\to[4,\infty)$ given by $f(x)=x^2+4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y)=\sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

OR

Prove that the relation R on the set Z of all integers defined by $(a,b)\in R \Leftrightarrow a-b$ is divisible by 5 is an equivalence relation.

12. Prove that: $2\tan^{-1}\frac{1}{5}+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)+2\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$.

13. Using properties of determinants, show that:

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca).$$

14. Show that the function $f(x)=|x+2|$ is continuous at every $x\in R$ but fails to be differentiable at $x=-2$.

15. Find $\frac{dy}{dx}$, if $y=(x)^{\cos x}+(\log x)^{x}$.

OR

If $y=\sqrt{\frac{1-x}{1+x}}$, show that $(1-x^2)\frac{dy}{dx}+y=0$.

16. Find the equation of the normals to the curve $y=x^3+2x+6$ which are parallel to the line $x+14y+4=0$.

17. Evaluate the following: $\int\frac{(3\sin\theta-2)\cos\theta}{5-\cos^2\theta-4\sin\theta}d\theta$

OR

Evaluate the following: $\int\frac{2+\sin 2x}{1+\cos 2x}e^x\,dx$

18. Using the properties of definite integral, evaluate the following: $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$.

19. Solve the following differential equation:

$$\frac{dy}{dx}+\frac{2x}{x^2+1}y=\frac{1}{(x^2+1)^2}\ ;\ y(0)=0$$

OR

Solve the following differential equation:

$$(x^3+y^3)\,dy-x^2y\,dx=0$$

20. Find the area of the parallelogram whose diagonals are determined by the vectors $\vec{a}=2\hat{i}-\hat{j}+k$ and $\vec{b}=3\hat{i}+4\hat{j}-k$.

21. Determine whether or not the following pair of lines intersect. If these intersects, then find the point of intersection, otherwise obtain the shortest distance between them:

$$\vec{r}=(\hat{i}+\hat{j}-k)+\lambda(3\hat{i}-\hat{j}) \text{ and } \vec{r}=(4\hat{i}-k)+\mu(2\hat{i}+3k).$$

22. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

SECTION – C

23. Using matrix method, solve the following system of linear equations:

$$x+2y+z=7 \ ; \ x+3z=11 \ ; \ 2x-3y=1$$

24. An Apache helicopter of enemy is flying along the curve given by $y=x^2+7$. A soldier, is placed at $(3,7)$, wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

OR

Show that the right circular cylinder of given volume open at the top has minimum total surface area, provided its height is equal to the radius of its base.

25. Evaluate the following integral as limit of sums: $\int_0^2 (x^2+2x+1)\,dx$

26. Using integration, find area of ΔABC whose vertices have coordinates $A(2,0), B(4,5)$ and $C(6,3)$.

27. Find the equation of the plane which contains line of intersection of planes $\vec{r}.(\hat{i}+2\hat{j}+3k)-4=0$, $\vec{r}.(2\hat{i}+\hat{j}-k)+5=0$ and which is perpendicular to the plane $\vec{r}.(5\hat{i}+3\hat{j}-6k)+8=0$.

OR

Find the image of the point $(1,2,3)$ in the plane $x+2y+4z=38$. Also find the distance of the given point from the plane.

28. Let X denote the number of colleges where you will apply after your results and $P(X=x)$ denotes your probability of getting admission in x number of colleges. It is given that

$$P(X=x)=\begin{cases} kx & \text{if } x=0 \text{ or } 1 \\ 2kx & \text{if } x=2 \\ k(5-x) & \text{if } x=3 \text{ or } 4 \end{cases} ; \ k \text{ is a positive constant}$$

(a) Find the value of k.
(b) What is the probability that you will get admission in exactly two colleges?
(c) Find the mean and variance of the probability distribution.

29. A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is Rs. 30 while by selling one table the profit is Rs. 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as L.P.P. and solve it graphically.