VAISHALI EDUCATION POINT

(Quality Education Provider)
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Sample Paper – 2009
Class – XII
Subject – MATHEMATICS

Time allowed: 3 hrs M.Marks:100

General Instructions:

1. All questions are compulsory
2. Q 1 – 10 carries 1 marks, Q 11 – 22 carries 4 marks Q-23to 29 carries 6 marks

SECTION – A
(1 mark each)

1. **Let** '*' be a binary operation on set Q - {1} defined by a * b = a + b – ab ; a, b ∈ Q - {1}. Find the identity element with respect to * on Q.

2. Find the principal values of $\operatorname{cosec}^{-1}2$ and $\operatorname{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)$.

3. If A = $\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, find the value of α for which $A^2 = B$.

4. By using the elementary row transformation, find the inverse of the matrix A = $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$.

5. Using determinants, find the value of k so that the points (k, 2- 2k), (-k+1, 2k) and (-4-k, 6-2k) may be collinear.

6. For what value of 'k ', the function:

$$f(x) = \begin{cases} \dfrac{\sqrt{5x+2}-\sqrt{4x+4}}{x-2} & if\ \chi \neq 2 \\ k, & \text{if } \chi = 2. \end{cases}$$ is continuous at x = 2 ?

7. Evaluate $\int e^{3\log x} x^4 dx.$

8. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

9. Given $|\vec{a}| = 10, \quad |\vec{b}| = 2, \quad \vec{a}.\vec{b} = 12, \quad \text{find } |\vec{a} \times \vec{b}|$.

10 The Cartesian equation of a line are 6x – 2 = 3y + 1 = 2z – 2. Find its direction ratios. Also write the vector equation of the line.

SECTION-B

(4 marks each)

11. Let N denote the set of all natural numbers and R be the relation N x N defined by (a,b) R (c,d) iff ad(b + c) = bc(a+d). Check whether R is an equivalence relation.

OR

Show that f: R - {-1} → R - {1} given by f(x) = $\dfrac{x}{x+1}$ is invertible. Also find f^{-1}.

12. Prove that $\tan^{-1}\left(\dfrac{63}{16}\right) = \sin^{-1}\left(\dfrac{5}{13}\right) + \cos^{-1}\left(\dfrac{3}{5}\right)$.

13. Using determinants, prove that

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2).$$

OR

If A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then using principle of mathematical induction prove that

$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ for every positive integer n.

14. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} |x| + 3, & \text{if } x \le -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \ge 3 \end{cases}$$

15. If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

OR

If x = a sec θ, y = b tan θ, prove that $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$.

16. Two equal sides of an isosceles triangle with fixed base 'b' are decreasing at the rate of 3 cm/sec. How fast is the area decreasing when the two equal sides are equal to the base.

17. Evaluate

$$\int \frac{x^2+5x+3}{x^2+3x+2}dx$$

18. Evaluate the following integral as limit of sum: $\int_0^2 (x^2+3)dx.$

19. Evaluate $\int_0^{\pi/4} \log(1+\tan x)dx.$

20. A girl walks 4 Km towards east, then she walks 3 Km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

21. Find the equation of the plane through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane 2x – 5y = 15.

22. From a lot of 10 items containing 3 defective, a sample of 4 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the sample is drawn without replacement, find (i) Mean of X (ii) Variance of X .

OR

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be hearts . Find the probability of the missing card to be a heart.

SECTION-C

(6 marks each)

23. Find A^{-1}, when A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence, solve the following system of equations:

x + 2y - 3z = -4
2x + 3y + 2z = 2
3x – 3y - 4z = 11.

24. Let AP and BQ be two vertical poles at points A and B respectively. If AP = 16 m. BQ = 22 m and AB = 20 m, then find the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum.

OR

The combined resistance R of two resistors R_1 and R_2 (R_1, R_2 >0) is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.

If $R_1 + R_2 = C$ (a constant). Show that the maximum resistance R is obtained by choosing $R_1 = R_2$.

25. Using integration, find the area of the region bounded by the following curves, after making a rough sketch: $y = 1 + |x+1|$; $x = -3$; $x = 3$; $y = 0$.

26. Solve the initial value problem:

$$(1 + y^2)\, dx = (\tan^{-1}y - x)\, dy;\ y(0) = 0.$$

27. Find the shortest distance between the following pairs of lines whose cartesian equations are:

$$\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2} \quad \text{and} \quad \frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-2}.$$

28. If an old man rides his motor cycle at 25 km/hr, he has to spend Rs 2 per Km on petrol. If he rides at a faster speed of 40 km/hr, the petrol cost increases to Rs 5 per Km. He has Rs 100 to spend on petrol and wishes to find maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it graphically.

29. There are two bags. The first bag contains 5 white and 3 black balls and the second bag contains 3 white and 5 black balls. Two balls are drawn at random from the first bag and are put into the second bag without noticing their colours. Then two balls are drawn from the second bag. Find the probability that the balls are white and black.

OR

If a fair coin is tossed 10 times, find the probability of

(i) exactly six heads.
(ii) at least six heads.
(iii) at most six heads.