VAISHALI EDUCATION POINT

(Quality Education Provider)
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Sample Paper – 2009
Class – XII
Subject – MATHEMATICS

Time : 3 hrs **Max. Marks : 100**

General Instructions:

1 All questions are compulsory
2 Q 1 – 10 carries 1 marks, Q 11 – 22 carries 4 marks Q-23to 29 carries 6 marks

SECTION – A

1. Show that * : $\mathbf{R} \times \mathbf{R} \to \mathbf{R}$ given by (a, b) → a + 4b² is a binary operation.

2. Find the value of cos (sec⁻¹ x + cosec⁻¹ x), | x | ≥ 1

3. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Find x and y Such that $A^2 = xA + yI$.

4. Find values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

5. Simplify : $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

6. Evaluate : $\int \frac{\sec^2(\log x)}{x} dx$.

7. Evaluate $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

8. Find $|\vec{a} - \vec{b}|$ if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2$; $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$

9. Find the direction cosines of x-axis.

10. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$, are such that $\vec{a} + \lambda \vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

SECTION-B

11. Let A = $\mathbf{R}$ – {3} and B = $\mathbf{R}$ – {1}. Consider the function f: A → B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$.
Show that f is bijective.

12. Using properties of determinants, prove that :

$$\begin{vmatrix} 1+a^2 & ab & ac \\ ab & 1+b^2 & bc \\ ac & bc & 1+c^2 \end{vmatrix} = (1+a^2+b^2+c^2)$$

13. Prove that : $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, \; -\frac{1}{\sqrt{2}} \le x \le 1$

OR

Solve for x : $\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\left(\frac{33}{36}\right)$.

14. If the function $f(x) = \begin{cases} 1 : x \le 3 \\ ax+b; 3 < x < 5 \\ 7; x \ge 5 \end{cases}$ is continuous at x = 3 and x = 5, then find the value of a & b

15. If $y = \frac{\log x}{x}$, Show that $\frac{d^2y}{dx^2} = \frac{2\log x - 3}{x^3}$

OR

If $y = (\sin^{-1}x)^2$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$

16. Find the intervals in which the function f(x) = sin 3x; $0 \le x \le \frac{\pi}{2}$

(i) is increasing (ii) is decreasing

17. Evaluate : $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$

OR

Evaluate : $\int_1^3 (x^2+5x+1)\, dx$ as a limit of a sum

18. Solve the following differential equation : $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

19. Find the particular solution of the differential equation $(1-x^2)\frac{dy}{dx} - xy = x^2$, given that y(0) = 2.

20. If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$, $\vec{\alpha} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

21. Find the equation of the plane that contains the point (1, – 1, 2) and is perpendicular to each of the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$.

22. A football match may be either won, drawn or lost by the host country's team, So there are three ways of forecasting the result of any one match, one correct and two

incorrect. Find the probability of forecasting at least three correct results for four matches.

SECTION-C

23. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence, Solve the system of linear equations :

$x + 2y - 3z = -4$, $2x + 3y + 2z = 2$, and $3x - 3y - 4z = 11$

OR

Using elementary row transformations, find inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

24. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius r is $\frac{2r}{\sqrt{3}}$.

OR

A given quantity of metal is to be cast into a solid half circular cylinder (i.e., with rectangular base and semicircular ends). Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its circular ends is $\pi : (\pi + 2)$

25. Evaluate: $\int \sqrt{\cot x} dx \, dx$.

26. Sketch the region enclosed between circles $x^2 + y^2 = 1$ and $x^2 + (y - 1)^2 = 1$. Also, find the area of region using integration.

OR

Sketch the region common to the circle $x^2 + y^2 = 16$ and the parabola $x^2 = 6y$. Also find the area of the region using integration

27. Find the foot of the perpendicular from P(1, 2, 3) on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, obtain the equation of the plane containing the line and the point (1, 2, 3).

28. Every gram of wheat provides 0.1 gm of proteins and 0.25 gm of carbohydrates. The corresponding values for rice are 0.05 gm and 0.5 gm respectively. Wheat costs Rs. 4 per kg and rice Rs. 6 per kg. The minimum daily requirements of proteins and carbohydrates for an average child are 50 gms and 200 gms respectively. In what quantities should wheat and rice be mixed in the daily diet to provide minimum daily requirements of proteins and carbohydrates at minimum cost. Frame an L.P.P. and solve it graphically.

29. A candidate has to reach the examination centre in time. Probability of him going by bus or scooter or by other means of transport are $\frac{3}{10}, \frac{1}{10}$ and $\frac{3}{5}$ respectively. The probability that he will be late is $\frac{1}{4}$ and $\frac{1}{3}$ respectively, if he travels by bus or scooter. But he reaches in time if he uses any other mode of transport. He reached late at the centre. Find the probability that he travelled by bus.