

VAISHALI EDUCATION POINT

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Class :- XII

Subject :- MATH

General Instructions

QNo.	Questions
1	$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix}$ <p>If then find the matrix X for which $A + B - X = 0$</p>
2	$\text{If } A = \begin{bmatrix} 1 & 2 & 0 \\ -7 & 2 & 6 \end{bmatrix}, B = \begin{bmatrix} 6 & 9 & 5 \\ -3 & 2 & 7 \end{bmatrix} \text{ and } C = \begin{bmatrix} 4 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix},$ <p>then show that $(A + B) + C = A + (B + C)$.</p>
3	$\text{If } A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ <p>then show that $A^2 = A$.</p>
4	<p>Find a single matrix expressing $\frac{1}{2} \begin{bmatrix} 1 & 3 \\ 1 & -4 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 8 & 4 \\ 4 & 8 \end{bmatrix}$.</p>
5	<p>Given that $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, show by induction, $A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$.</p>
6	$\text{If } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix},$ <p>verify that $(A + B)^2 = A^2 + B^2 + 2AB$.</p>
7	$\text{If } A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ <p>Prove that $A^3 - 6A^2 + 7A + 2I = 0$</p>
8	$\text{Show that the matrix } A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 5 \end{bmatrix}$ <p>is a symmetric matrix.</p>
9	$\text{Show that } A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix}$ <p>is a skew symmetric matrix.</p>
10	<p>Express the matrices as the sum of a symmetric and a skew symmetric matrix. $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$</p>
11	<p>Express the matrices as the sum of a symmetric and a skew symmetric matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.</p>
12	$\text{If } A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix},$ <p>verify that $A + A'$ is a symmetric matrix</p>
13	$\text{If } A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix},$ <p>verify that $A - A'$ is a skew symmetric matrix.</p>

14

If $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 9 & 1 & 0 \end{bmatrix}$, verify that: $A + A'$ is a symmetric matrix

15

$$A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$$

In the matrix, write: (i) The order of the matrix (ii) The number of elements, (iii) Write the elements a_{13} , a_{21} , a_{33} , a_{24} , a_{23}

16

If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

17

If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

18

Construct a 3×4 matrix, whose elements are given by (i) $a_{ij} = \frac{1}{2}|-3i + j|$ (ii) $a_{ij} = 2i - j$

19

Find the value of x, y, and z from the following equation:

$$(i) \begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix} \quad (ii) \begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

$$(iii) \begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

20

Find the value of a, b, c, and d from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

21

$A = [a_{ij}]_{m \times n}$ is a square matrix, if (A) $m < n$ (B) $m > n$ (C) $m = n$ (D) None of these

22

Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

$$(A) \quad x = \frac{-1}{3}, y = 7$$

$$(B) \text{ Not possible to find } (C) \quad y = 7, x = \frac{-2}{3}$$

$$(D) \quad x = \frac{-1}{3}, y = \frac{-2}{3}$$

23

The number of all possible matrices of order 3×3 with each entry 0 or 1 is: (A) 27 (B) 18 (C) 81 (D) 512

24

Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$ Find each of the following (i) $A+B$ (ii) $A-B$ (iii) $3A-C$ (iv) AB (v) BA

25

Compute the following:

$$(i) \begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} \quad (ii) \begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$$

$$(iii) \begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

$$(v) \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

26

Compute the indicated products

$$(i) \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2 \quad 3 \quad 4]$$

$$(iii) \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$(iv) \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$(v) \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

$$(vi) \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$

27

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix},$$

If then compute $(A+B)$ and $(A-B)$. Also, verify that $A+(B-C)=(A+B)-C$.

28

$$A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$$

If then compute $3A-5B$.

29

$$\text{Simplify } \cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$$

30

$$\text{Find } X \text{ and } Y, \text{ if (i) } X+Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \text{ and } X-Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

31	(ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$
32	Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$
33	Find x and y, if $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$
34	Solve the equation for x, y, z and t if $2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$
35	If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find values of x and y.
36	Given $3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x, y, z and w.
37	If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y) = F(x+y)$
38	Show that (i) $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$
39	(ii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$
40	Find $A^2 - 5A + 6I$ if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$
41	If $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and $B = [1 \ 0 \ 4]$ find (AB) . (2007)
42	Find the area of the triangle whose vertices are (2, 7), (1, 1) and (10, 8). (2007)
43	If $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$, show that $A^2 - 6A + 17I = 0$. Hence find A^{-1} . (2007)
44	Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{i}{j}$. (2008 Comp.)
45	If $\begin{bmatrix} x + 2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$, find the values of x and y. (2008 Comp.)
46	Find the value of x and y if: $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ (2008)
47	Find the co-factor of a_{12} in the following: $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ (2008)

46

$$A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as sum of two matrices such that one is symmetric and the other is skew symmetric.

OR

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, verify that $A^2 - 4A - 5I$ (2008)

47

$$A = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$$

If $A = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$, find $A + A'$, where A' is the transpose of matrix A. (2009 Comp.)

48

$$\begin{bmatrix} 15 & x+y \\ 2 & y \end{bmatrix} = \begin{bmatrix} 15 & 8 \\ x-y & 3 \end{bmatrix}$$

If $\begin{bmatrix} 15 & x+y \\ 2 & y \end{bmatrix} = \begin{bmatrix} 15 & 8 \\ x-y & 3 \end{bmatrix}$, find the value of x. (2009 Comp.)

49

If matrix $A = (1 \ 2 \ 3)$, write AA' ; where A' is the transpose of matrix A. (2009)

50

$$\begin{pmatrix} a+b & 2 \\ 5 & b \end{pmatrix} = \begin{pmatrix} 6 & 5 \\ 2 & 2 \end{pmatrix}$$

If $\begin{pmatrix} a+b & 2 \\ 5 & b \end{pmatrix} = \begin{pmatrix} 6 & 5 \\ 2 & 2 \end{pmatrix}$, then find a. (2010 Comp.)

51

If A is a matrix of order 3×4 and B is a matrix of order 4×3 , find the order of the matrix (AB). (2010 Comp.)

52

$$A = \begin{pmatrix} 3 & 1 \\ 2 & -3 \end{pmatrix}$$

If $A = \begin{pmatrix} 3 & 1 \\ 2 & -3 \end{pmatrix}$, then find $|\text{adj} A|$ (2010 Comp.)

53

$$A = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$$

If $A = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$, then for what value of α is A an identity matrix? (2010)

54

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ k & 23 \end{pmatrix}$$

If $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ k & 23 \end{pmatrix}$, then write the value of k. (2010)

55

Using elementary row operations, find the inverse of the following matrix: $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ (2010)

56

$$\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$$

For what value of x is the matrix $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ singular? (2011 Comp.)

57

$$\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$$

If $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$, then find the value of y. (2011 Comp.)

58

For a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by $a_{ij} = \frac{i}{j}$ write the value of a_{12} . (2011)

59

$$\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$$

For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular? (2011)

60

Using elementary transformations, find the inverse of the matrix. $\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$ (2011)

61

Construct a 2×2 matrix $A = [a_{ij}]$, where $a_{ij} = \frac{(3i-j)^2}{2}$.

62

If a matrix has 8 elements, list the possible orders of a matrix.

63

Construct a 3×1 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by, $a_{ij} = \frac{1}{2} |-3i + j|$

64

$$A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$$

If $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, then find the matrix X, of order 3×2 such that $2A + 3X = 5B$.

65

$$A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$$

If $A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$, then verify that $A'A = I$.

66

If A and B are symmetric matrices of the same order, then show that $AB - BA$ is a skew-symmetric matrix.

67

$$\text{Find X and Y, if } X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} \text{ and } X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}.$$

68

$$\text{Find x and y, if } \begin{pmatrix} x \\ 2y \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}.$$

69

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $B = [2 \ -3 \ 4]$, find AB.

70

If A, B and AB are symmetric matrices, then what is the relation between AB and BA.

71

$$\text{Find the values of x, y and z, if } \begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}.$$

72

$$\text{Find x and y, if } 2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}.$$

73

Represent the matrix $A = \begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrix.

74

$$\text{Find matrices A and B for the following: } A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}; 2A + 3B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}.$$

75

$$\text{Find the values of p and q such that } A^2 + pI = qA, \text{ where } A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

76

$$\text{If } A = \begin{bmatrix} 1 & 6 & 9 \\ 2 & 1 & 5 \end{bmatrix}^T \text{ and } B = \begin{bmatrix} 9 & 3 & 8 \\ 0 & 5 & -11 \end{bmatrix}, \text{ verify that } (AB)' = B'A'.$$

77

$$\text{Find the values of a, b, c, d from the following: } \begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}.$$

78

$$\text{If } A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix} \text{ and } B = [1 \ 3 \ -6], \text{ verify that } (BA)' = A'B'.$$

79

$$\text{If } A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}, \text{ show that } AB \neq BA.$$

80

$$\text{If } A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}, \text{ show that } A^3 - 23A - 40I = O.$$

81

Express the matrix $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as a sum of symmetric and a skew-symmetric matrix.

82

Find x , if $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$.

83

If $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$.

84

If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$, for $n \in \mathbb{N}$.

85

If $A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, find $(A + 2B)^2$.

86

Find the matrix X such that, $\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix} X = \begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$.

87

If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then prove that for $n \in \mathbb{N}$, $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$.

88

Using elementary transformations, find the inverse of the following matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$.

89

Find the inverse using elementary transformation, if exists, for the matrix $\begin{bmatrix} 8 & -4 \\ -2 & 1 \end{bmatrix}$.

90

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$, for $n \in \mathbb{N}$.

91

Find the inverse of matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary transformations.

92

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$, use the result to find A^4 .

93

If $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$, compute $A^2 B^2$.

94

A trust fund has Rs. 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year and the second bond pays 7% interest per year. Using matrix multiplication, find how to divide Rs. 30,000 among two types of bonds, if the trust fund must obtain an annual total

95	<p>interest of (i) Rs. 1,800 (ii) Rs. 2,000.</p> <p>Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^n I + na^{n-1}bA$, where I is the identity matrix of order 2 and $n \in \mathbb{N}$.</p>
96	<p>If $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, find x, y, z, w.</p>