

**VAISHALI EDUCATION POINT**  
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**CONTINUITY & DIFFERENTIABILITY**

**Class :- XII      Subject :- MATH**

**General Instructions**

QNo.	Questions
1	Examine the following functions for continuity. (a) $f(x) = x - 5$ (b) $f(x) = \frac{1}{x-5}, x \neq 5$ (c) $f(x) = \frac{x^2 - 25}{x+5}, x \neq -5$ (d) $f(x) =  x - 5 $
2	$f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$ Is the function f defined by continuous at x = 0? At x = 1? At x = 2?
3	Find all points of discontinuity of f, where f is defined by $f(x) = \begin{cases} x^3 - 3, & \text{if } x \leq 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases}$
4	Find all points of discontinuity of f, where f is defined by $f(x) = \begin{cases} x^{10} - 1, & \text{if } x \leq 1 \\ x^2, & \text{if } x > 1 \end{cases}$
5	Discuss the continuity of the function f, where f is defined by $f(x) = \begin{cases} 3, & \text{if } 0 \leq x \leq 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \leq x \leq 10 \end{cases}$
6	Discuss the continuity of the function f, where f is defined by $f(x) = \begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \leq x \leq 1 \\ 4x, & \text{if } x > 1 \end{cases}$
7	Discuss the continuity of the following functions. (a) $f(x) = \sin x + \cos x$ (b) $f(x) = \sin x - \cos x$ (c) $f(x) = \sin x \times \cos x$
8	Find the points of discontinuity of f, where

$$f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ x+1, & \text{if } x \geq 0 \end{cases}$$

9 Determine if f defined by

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

is a continuous function?

10 Examine the continuity of f, where f is defined by

$$f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1 & \text{if } x = 0 \end{cases}$$

11

Find the value of K if the function  $f(x) = \begin{cases} kx^2, & x \geq 1 \\ 4, & x < 1 \end{cases}$  is continuous at  $x = 1$ . **(2007)**

12

Discuss the continuity of the function  $f(x) = \begin{cases} x+1 & \text{if } x \geq 1 \\ x^2+1 & \text{if } x < 1 \end{cases}$  at  $x = 1$  **(2007)**

13

Differentiate  $\sin(x^2+1)$  with respect to x from first principle. **(2007)**

14

If  $y = \sin(\log x)$ , prove that  $x^2 \cdot \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ .  
OR

Verify Rolle's theorem for the function :  $f(x) = x^2 - 5x + 4$  on  $[1, 4]$  **(2007)**

15

Differentiate  $\sin \sqrt{2x}$  with respect to x from first principle. **(2007)**

16

If the following function  $f(x)$  is continuous at  $x = 0$ , find the value of k:

$$f(x) = \begin{cases} \frac{1 - \cos 2x}{2x^2} & x \neq 0 \\ k & x = 0 \end{cases} \quad \text{(2008)}$$

17

$$\text{If } y = (\log x)^{\cos x} + \frac{x^2+1}{x^2-1},$$

OR

If  $x = a \left( \cos t + \log \tan \frac{t}{2} \right)$  and  $y = a \sin t$ , find  $\frac{dy}{dx}$  **(2008)**

18

For what value of k is the following function continuous at  $x = 2$ ?

$$f(x) = \begin{cases} 2x+1 & ; x < 2 \\ k & ; x = 2 \\ 3x-1 & ; x > 2 \end{cases} \quad \text{(2008)}$$

19

Differentiate the following with respect to x :  $\tan^{-1} \left( \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right)$  **(2008)**

20

$$f(x) = \begin{cases} 2x-1, & x < 2 \\ a, & x = 2 \\ x+1, & x > 2 \end{cases}$$

If the function defined by  $f(x)$  is continuous at  $x = 2$ , find the value of  $a$ . Also discuss the continuity of  $f(x)$  at  $x = 3$  **(2009 Comp.)**

21

If  $y = (\log x)^x + (x)^{\cos x}$ , find  $\frac{dy}{dx}$ .

OR

If  $x = a \left( \cos \theta + \log \tan \frac{\theta}{2} \right)$  and  $y = a \sin \theta$ , find the value of  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{4}$  **(2009 comp.)**

22

Differentiate the following function w. r. t.  $x$ :  $x^{\sin x} + (\sin x)^{\cos x}$  **(2009)**

23

Find  $\frac{dy}{dx}$  if  $(x^2 + y^2)^2 = xy$

OR

If  $y = 3 \cos(\log x) + 4 \sin(\log x)$ , then show that  $x^2 \cdot \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$  **(2009)**

24

For what value of  $k$  is the function defined by  $f(x) = \begin{cases} k(x^2 + 2), & \text{if } x \leq 0 \\ 3x + 1, & \text{if } x > 0 \end{cases}$  continuous at  $x = 0$ ? Also write whether the function is continuous at  $x = 1$ . **(2010 Comp.)**

25

Show that the function  $f(x)$  defined as follows, is continuous at  $x = 2$ , but not

$$f(x) = \begin{cases} 3x-2, & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x \leq 2 \\ 5x-4, & x > 2 \end{cases}$$

differentiable there at :  $x = 2$

OR

Find  $\frac{dy}{dx}$ , if  $y = \sin^{-1} [x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}]$  **(2010)**

26

Discuss the continuity of the function  $f(x)$  at  $x = \frac{1}{2}$  when  $f(x)$  is defined as

$$f(x) = \begin{cases} \frac{1}{2} + x, & 0 \leq x < \frac{1}{2} \\ 1, & x = \frac{1}{2} \\ \frac{3}{2} + x, & \frac{1}{2} < x \leq 1 \end{cases}$$

follows : **(2011 Comp.)**

27

If  $x\sqrt{1+y} + y\sqrt{1+x} = 0, (x \neq y)$ , then prove that  $\frac{dy}{dx} = \frac{1}{(1+x)^2}$

OR

If  $x = a \left( \cos t + \log \tan \frac{t}{2} \right)$  and  $y = a \sin t$ , find  $\frac{dy}{dx}$ . **(2011 Comp.)**

28

$$f(x) = \begin{cases} a \sin \frac{\pi}{2}(x+1), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases} \text{ is}$$

Find the value of 'a' for which the function f defined as continuous at  $x = 0$ . **(2011)**

29

Differentiate  $x^x \cos x + \frac{x^2+1}{x^2-1}$  w.r.t. x

OR

If  $x = a(\theta - \sin \theta)$ ,  $y = a(1 + \cos \theta)$ , find  $\frac{d^2y}{dx^2}$  **(2011)**

30

Examine the continuity of the function  $f(x) = x^2 + 5$  at  $x = -1$ .

31

Examine the continuity of the function  $f(x) = \frac{1}{x+3}$ ,  $x \in \mathbb{R}$ .

32

Find the value of m, for which the function  $f(x) = \begin{cases} m(x^2 - x) & x > 0 \\ \cos x & x \leq 0 \end{cases}$  is continuous at  $x = 0$ .

33

If function  $f(x) = \frac{2x+3 \sin x}{3x+2 \sin x}$ , for  $x \neq 0$  is continuous at  $x = 0$ , then find  $f(0)$ .

34

Show that the function  $f(x) = \begin{cases} x^3 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$  is not continuous at  $x = 0$ .

35

Find the point of discontinuity, if any, for the function  $f(x) = \frac{1}{x-5}$ .

36

State the points of discontinuity for the function  $f(x) = [x]$  in  $-3 < x < 3$ .

37

Is the function  $f(x) = \frac{3x+4 \tan x}{5x}$  continuous at  $x = 0$ ? Give reasons.

38

Discuss the differentiability of the function  $f(x) = (x-1)^{2/3}$  at  $x = 1$ .

39

Find the derivative of,  $e^{\sqrt{x}+3}$ , with respect to x.

40

Differentiate the following w.r.t. x,  $y = 5^{\log(\sin x)}$ .

41

Find  $\frac{dy}{dx}$ , when  $\sqrt{x} + \sqrt{y} = 5$  at (4, 9).

42

If  $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$ , find  $\frac{dy}{dx}$ .

43

Given  $f(0) = -2$ ,  $f'(0) = 3$ . Find  $h'(0)$  where  $h(x) = x f(x)$ .

44

Verify LMV theorem for the function  $y = \sqrt{x-2}$  in [2, 6].

45

Find the derivative of  $\sin^{-1}\left(\frac{1-x}{1+x}\right)$ , w.r.t.  $\sqrt{x}$ .

46

If  $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ , show that  $\frac{dy}{dx} - \sec x = 0$ .

47

If  $e^x + e^y = e^{x+y}$ , prove that  $\frac{dy}{dx} = -e^{y-x}$

48

Show that the function 'f' defined by  $f(x) = |1 - x + |x||$ , where x is any real number, is a continuous function.

49

Prove that  $\frac{d}{dx} \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right] = \sqrt{a^2 - x^2}$

50

If  $y = a \cos(\log x) + b \sin(\log x)$ , prove that  $x^2 y_2 + x y_1 + y = 0$ .

51

If  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$ , find  $\frac{d^2 y}{dx^2}$  at  $\theta = \frac{\pi}{2}$ .

52

Find the derivative of  $\log(\sqrt{\sin x - \cos x})$ , w.r.t.  $\sin x$ .

53

If  $y = (\cot^{-1} x)^2$ , show that  $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 - 2 = 0$ .

54

Find the values of a and b, such that the function defined by

$f(x) = \begin{cases} 5, & x \leq 2 \\ ax + b, & 2 < x < 10 \\ 21, & x \geq 10 \end{cases}$  is a continuous function.

55

Differentiate w.r.t. x :  $\log \sqrt{\frac{1 - \sin x}{1 + \sin x}}$ .

56

If  $y = \sin \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\}$ . then prove that  $\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$ .

57

If  $(a + bx) \cdot e^{y/x} = x$ , then prove that  $x^3 \frac{d^2 y}{dx^2} = \left( x \frac{dy}{dx} - y \right)^2$ .

58

If  $y = x^{x^{x^{\dots \infty}}}$ , then prove that  $\frac{dy}{dx} = \frac{y^2}{x(1 - y \log x)}$ .