PHYSICS CLASS XI
CHAPTER – 9 MECHANICAL PROPERTIES OF SOLIDS

Q.1. What is the Young's modulus for a perfect rigid body?

Ans. Young's modulus (Y) = $\frac{F}{A} \times \frac{l}{\Delta l}$

For a perfectly rigid body, change in length Δl = 0

∴ Y = $\frac{F}{A} \times \frac{l}{0} = \infty$

Therefore, Young's modulus for a perfectly rigid body is ∞ .

Q.2. What is Bulk modulus for a perfectly rigid body?

Ans. Bulk modulus (B) = $\frac{P}{\Delta V/V} = \frac{pV}{\Delta V}$

For perfectly rigid body, change in volume ΔV = 0

∴ B = $\frac{pV}{0} = \infty$

Therefore, Bulk modulus for a perfectly rigid body is ∞.

Q.3. A metal bar of length L, area of cross-section A, Young's modulus Y and coefficient of linear expansion ∝, is clamped between two stout pillars. What is the force exerted by the bar when it is heated through t°C?

Ans. Y = $\frac{FL}{Al}$, where l = L ∝ Δt and l = change in length.

Y = $\frac{FL}{AL\propto\Delta t} = \frac{F}{A\propto\Delta t} = \frac{F}{A\propto\Delta t}$

Address: Plot No 420, Behind Shopprix Mall, Vaishali Sector 5, Ghaziabad – 201010
M: 9999907099, 9818932244 O: 0120-4130999
Website: www.vaishalieducationpoint.com , www.educationsolution.co

Q.4. A wire increases by 10^{-3} of its length when a stress of 10^{8} Nm^{-2} is applied to it. What is the Young's modulus of the material of the wire?

Ans. Given, $\Delta L = 10^{-3}L$, with L as the original length

Strain = $\frac{\Delta L}{L} = 10^{-3}$L and Stress = $\frac{F}{A}$ = 10^{8} N/m^2

$$\therefore \quad Y = \frac{Stress}{Strain} = \frac{F/A}{\Delta L/L}$$

$$Y = \frac{1\times10^8}{10^{-3}} = 10^{11} N/m^2$$

Q.5. Two wires A and B are of the same material. Their lengths are in the ratio 1:2 and the diameters in the ratio 2:1 If they are pulled by the same dorce, what will be the ratio of their increase in lengths?

Ans. We know $\Delta L = \frac{FL}{AY}$, $\frac{L_A}{L_B} = \frac{1}{2}$ (given)

[∴ The wires A and B are pulled by the same force and they are made up of same material hence, $F_A = F_B = F$, $Y_A = Y_B = Y$]

$$\frac{\Delta L_A}{\Delta L_B} = \frac{L_A}{\pi r_A^2} \times \frac{\pi r_B^2}{L_B}, \frac{r_A}{r_B} = \frac{2}{1}$$

$$\frac{\Delta L_A}{\Delta L_B} = \frac{L_A}{L_B} \times \left(\frac{r_B}{r_A}\right)^2$$

$$\frac{\Delta L_A}{\Delta L_B} = \frac{1}{2} \times \left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

$$\frac{\Delta L_A}{\Delta L_B} = \frac{1}{8}$$

Q.6. A steel rod ($Y = 2.0 \times 10^{11} N/m^2$ and $\propto = 10^{-5} C^{-1}$) of length 1 m and area of cross – section 1 cm^2 is heated from 0°C to 200°C, without being allowed to extend or bend. What is the tension produced in the rod?

Address: Plot No 420, Behind Shopprix Mall, Vaishali Sector 5, Ghaziabad – 201010
M: 9999907099, 9818932244 O: 0120-4130999
Website: www.vaishalieducationpoint.com , www.educationsolution.co

Ans. Given, Young's modulus of steel

$$Y = 2.0 \times 10^{11}\ N/m^2$$

Coefficient of thermal expansion $\propto = 10^{-5\circ}\ C^{-1}$

Length l = 1 m

Area of cross section A = $1 cm^2 = 1 \times 10^{-4}\ m^2$

Increase in temperature Δt = 200°C - 0°C = 200°C

Tension produced in steel rod (F) = YA $\propto \Delta t$

$= 2.0 \times 10^{11} \times 1 \times 10^{-4} \times 10^{-5} \times 200$

$= 4 \times 10^4$ N

Q.7. To what depth must a rubber ball be taken in deep sea so that its volume is decreased by 0.1%. (The Bulk modulus of rubber is 9.8 × 10^8 N/m^2 ; and the density of sea water is 10^3 kg/m^3.)

Ans. Bulk modulus of rubber (B) = $9.8 \times 10^8\ N/m^2$

Density of sea water (ρ) = $10^3 kg/m^3$

Percentage decrease in volume,

$\left(\frac{\Delta V}{V} \times 100\right) = 0.1$ or $\frac{\Delta V}{V} = \frac{0.1}{100}$

or $\frac{\Delta V}{V} = \frac{1}{1000}$

Let the rubber ball be taken up to depth h.

∴ Change in pressure (p) = h$\rho\gamma g$

∴ Bulk modulus (B) = $\frac{p}{(\Delta V/V)} = \frac{hpg}{(\Delta V/V)}$

Address: Plot No 420, Behind Shopprix Mall, Vaishali Sector 5, Ghaziabad – 201010
M: 9999907099, 9818932244 O: 0120-4130999
Website: www.vaishalieducationpoint.com , www.educationsolution.co

or $h = \frac{B\times(\Delta V/V)}{\rho g} = \frac{9.8\times10^8\times\frac{1}{1000}}{10^3\times9.8}$ = 100 m

Q.8. The Marina trench is located in the Pacific ocean and at one place, it is nearly 11 km beneath the surfaceof water. The water pressure at the bottom of the trench is about 1.1 × 10^8 Pa. A steel ball of initial volume 0.32 m^3 is dropped into the ocean and falls to the bottom of trench. What is the change in the volume of the ball when it reaches to the bottom if the Bulk modulus of steel is 1.6 × $10^{11} N/m^2$?

Ans. Depth (h) = 11 km = 11 × 10^3 m

Pressure at the bottom of the trench (p) = 1.1 × 10^8 Pa

Initial volume of the ball (V) = 0.32 m^3

Bulk modulus of steel (B) = 1.6 × 10^{11} N/m^2

Bulk modulus of steel (B) = $\frac{P}{(\Delta V/V)} = \frac{pV}{\Delta V}$

$$\Delta V = \frac{pV}{B} = \frac{1.1\times10^8\times0.32}{1.6\times10^{11}}$$

$$= 2.2 \times 10^{-4}\ m^3$$

Q.9. How much should the pressure on a litre of water be changed to compress it by 0.10%. Bulk modulus of elasticity of water = 2.2 × 10^9 N/m^2.

Ans. Change in volume, $\Delta V = V \times \frac{0.10}{100}$

or $\frac{\Delta V}{V} = \frac{0.10}{100} = 1 \times 10^{-3}$

Bulk modulus of water (B) = 2.2 × 10^9 N/m^2

Pressure on water (Δp) = ?

Address: Plot No 420, Behind Shopprix Mall, Vaishali Sector 5, Ghaziabad – 201010
M: 9999907099, 9818932244 O: 0120-4130999
Website: www.vaishalieducationpoint.com , www.educationsolution.co

Bulk modulus of water (B) = $\frac{\Delta p}{\Delta V/V}$

or $\Delta p = B \times \frac{\Delta V}{V}$

$= 2.2 \times 10^9 \times 1 \times 10^{-3}$

$= 2.2 \times 10^6$ N/m^2

Q.10. A steel wire of length 4.7 m and cross – sectional area 3.0 × 10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area 4.0 × $10^{-5} m^2$ under a given load.

What is the ratio of the Young's modulus of steel to that of copper?

Ans. Given, for steel wire length (l_1) = 4.7 m

Area of cross – section (A_1) = 3.0 × 10^{-5} m^2

For copper wire

Length (l_2) = 3.5 m

Area of cross – section (A_2) = 4.0 × 10^{-5} m^2

Let F be the given load under which steel and copper wires be stretched by the same amount Δl.

Young's modulus (Y) = $\frac{F/A}{\Delta l/l} = \frac{F \times l}{A \times \Delta l}$

For steel, $Y_s = \frac{F \times l_2}{A_1 \times \Delta l}$ ...(i)

For copper $Y_c = \frac{F \times l_2}{A_2 \times \Delta l}$ .....(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{Y_s}{Y_c} = \frac{F \times l_1}{A_1 \times \Delta l} \times \frac{A_2 \times \Delta l}{F \times l_2}$$

Address: Plot No 420, Behind Shopprix Mall, Vaishali Sector 5, Ghaziabad – 201010
M: 9999907099, 9818932244 O: 0120-4130999
Website: www.vaishalieducationpoint.com , www.educationsolution.co

$= \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \frac{4.7}{3.5} \times \frac{4.0\times10^{-5}}{3.0\times10^{-5}}$

$\frac{Y_S}{Y_C} = \frac{18.8}{10.5} = 1.79 = 1.8$

Q.11. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7 × 10^6 Pa. Bulk modulus for copper = 140 × 10^9 Pa.

Ans. Given, each side of cube (l) = 10 cm = 0.1 m

Hydraulic pressure (p) = 7 × 10^6 Pa

Bulk modulus for copper (B) = 140 × 10^9 Pa

Volume contraction (ΔV) = ?

Volume of the cube (V) = l^3 = $(0.1)^3$

= 1 × 10^{-3} m^3

Bulk modulus for copper (B) = $\frac{p}{\Delta V/V}$

= $\frac{pV}{\Delta V}$

or $\Delta V = \frac{pV}{B}$

$\Delta V = \frac{7\times10^6\times1\times10^{-3}}{140\times10^9} = \frac{1}{20} \times 10^{-6}\ m^3$

= 0.05 × 10^{-6} m^3

= 5 × 10^{-8} m^3

Q.12. Identical springs of steel and copper are equally stretched. On which, more work will have to be done?

Ans. Work done in stretching a wire is given by

Address: Plot No 420, Behind Shopprix Mall, Vaishali Sector 5, Ghaziabad – 201010
M: 9999907099, 9818932244 O: 0120-4130999
Website: www.vaishalieducationpoint.com , www.educationsolution.co

$$W = \frac{1}{2} F \times \Delta l$$

As spring of steel and copper are equally stretched.

Therefore, for same force (F),

$$W \propto \Delta l$$

Young's modulus (Y) = $\frac{F}{A} \times \frac{l}{\Delta l}$

or $\Delta l = \frac{F}{A} \times \frac{l}{Y}$

As both springs are Identical,

$\therefore$ $\Delta l \propto \frac{1}{Y}$

From Eqs. (i) and (ii), we get $W \propto \frac{1}{Y}$

$\therefore$ $\frac{W_{steel}}{W_{copper}} = \frac{Y_{copper}}{Y_{steel}} < 1$ (As $Y_{steel} > Y_{copper}$)

or $W_{steel} < W_{copper}$

Therefore, more work will be done for stretching copper spring.

Q.13. Construction for metro line was carried out day and night. One night, when the work was in full swing, suddenly chain of the crane, lifting a heavy concrete block, snapped and it fell down. Immediately, people from nearby area came for help. They lifted the concrete and saved many lives. Injured were transferred to hospital without waiting for police to arrive.

(i) What values of locals helped in saving lives?

Address: Plot No 420, Behind Shopprix Mall, Vaishali Sector 5, Ghaziabad – 201010
M: 9999907099, 9818932244 O: 0120-4130999
Website: www.vaishalieducationpoint.com , www.educationsolution.co

(ii) A crane having steel ropes is used to lift heavy loads upto 10^4 kg. The elastic limit for steel is 3×10^8 Nm^{-2}. What should be the radius r of the steel rope used?

(iii) Which is more elastic – rubber or steel?

Ans. (i) Quick reaction of locals and their help without waiting for police to arrive helped saving lives.

(ii) Given m = 10^4 kg and elastic limit = 3×10^8 N/m^2

Ultimate stress = $\frac{load\ to\ be\ lifted}{area\ of\ cross-section\ of\ rope} = \frac{mg}{\pi r^2}$

$\Rightarrow 3 \times 10^8 = \frac{10^4 \times 10}{\frac{22}{7} \times r^2} \Rightarrow r = \left(\frac{7 \times 10^4 \times 10}{22 \times 3 \times 10^8}\right)^{1/2}$

= 0.0103 m = 1.03 cm

(iii) Steel is more elastic as Young's modulus for steel is more than Young's modulus of rubber. i.e., $Y_S > Y_R$.

Q.14. Ajay was a very naughty boy. One day, he bought a rubber cord catapult from market and started hitting passerby. When his father came to know about this, he immediately called Ajay and scolded him. He made him realize what damage his act could do. Ajay realized his mistake and apologised for his mistake.

(i) What values do you associate with Ajay?

(ii) A rubber cord catapult has a cross – section area of 1 mm^2 and total unstretched length 10 cm. It is stretched to 12 cm and then released to project a

Address: Plot No 420, Behind Shopprix Mall, Vaishali Sector 5, Ghaziabad – 201010
M: 9999907099, 9818932244 O: 0120-4130999
Website: www.vaishalieducationpoint.com , www.educationsolution.co

missile of mass 5 g. Taking Young's modulus for rubber as 5.0 × 10^8 Nm^{-2}, find the tension in the cord. Also, find the velocity of projection of the projectile.

(iii) Breaking force for a wire is F. What will be breaking forces for two parallel wires of the size?

Ans. (i) Ajay is naughty but when he is made to realize his mistake, he is ready to apologise.

(ii) Δl = 12 – 10 = 2 cm = 0.02 m, l = 10 cm = 0.1 m,

a = 1mm^2 = 10^{-6} m^2 and Y = 5 × 10^8 N/m^2

As, $Y = \frac{F}{a} \times \frac{l}{\Delta l}$

∴ $F = \frac{Ya\Delta l}{l} = \frac{5\times10^8\times10^{-6}\times0.02}{0.1}$ = 100 N

KE of missile = elastic potential energy.

Given m = 5 g,

From 1st equation we have tension = 100 N

$$\frac{1}{2}mv^2 = \frac{1}{2} \times tension \times extension$$

$$v = \sqrt{\frac{tension\times0.02}{m}} = \sqrt{\frac{100\times0.02}{5/1000}} = 20\ m/s$$

(iii) 2F is the breaking forces for two parallel wires of this size.

Volume of water at the surface, $V = \frac{m}{\rho}$

At the given depth, $V' = \frac{m}{\rho'}$

Address: Plot No 420, Behind Shopprix Mall, Vaishali Sector 5, Ghaziabad – 201010
M: 9999907099, 9818932244 O: 0120-4130999
Website: www.vaishalieducationpoint.com , www.educationsolution.co

∴ Change in volume, ΔV = V – V' = $m\left(\frac{1}{\rho}-\frac{1}{\rho'}\right)$

Volumetric strain = $\frac{\Delta V}{V}$

$= m\left(\frac{1}{\rho}-\frac{1}{\rho'}\right)\times\frac{\rho}{m}$

$= \left(1-\frac{\rho}{\rho'}\right)$

Compressibility = $\frac{1}{Bulk\ modulus\ (B)}$

$= \frac{1}{\frac{\Delta p}{(\Delta V/V)}} = \frac{\Delta V}{\Delta p V}$

$45.8 \times 10^{-11} = \left(1-\frac{\rho}{\rho'}\right)\times\frac{1}{80\times1.013\times10^5}$

$45.8 \times 10^{-11} \times 80 \times 1.013 \times 10^5 = 1-\frac{1.03\times10^3}{\rho'}$

$3.712 \times 10^{-3} = 1-\frac{1.03\times10^3}{\rho'}$

$\frac{1.03\times10^3}{\rho'} = 1 - 3.712 \times 10^{-3}$

or $\rho' = \frac{1.03\times10^3}{1-0.003712} = 1.034 \times 10^3 \text{kg}/m^3$

Q.15. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50000 kg. The inner and outer radii of each column are 30 cm and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. Young's modulus, Y = 2.0 × 10^{11} Pa.

Ans. Given, total mass supported by cylindrical columns (m)

= 50000 kg

∴ Total weight supported by cylindrical columns = mg = 50000 = 9.8 = 490000 N

Address: Plot No 420, Behind Shopprix Mall, Vaishali Sector 5, Ghaziabad – 201010
M: 9999907099, 9818932244 O: 0120-4130999
Website: www.vaishalieducationpoint.com , www.educationsolution.co

∴ Load acting on each cylindrical support

$$F = \frac{mg}{4} = \frac{490000}{4} N = 122500 \text{ N}$$

Inner radius of each column (r_1) = 30 cm = 0.3 m

Outer radius of each column (r_2) = 60 cm = 0.6 m

∴ Area of cross – section of each cylindrical column

$$A = \pi r_2^2 - \pi r_1^2 = \pi(r_2^2 - r_1^2)$$

$$= 3.14 [(0.6)^2 - (0.3)^2]$$

$$= 3.14 \times 0.27 \, m^2$$

Young's modulus (Y) = 2×10^{11} Pa

Compressional strain of each column = ?

Young' s modulus (Y) = $\frac{Compressional\ stress}{Compressional\ strain}$

or Compressional strain = $\frac{Compressional\ stress}{Young's\ modulus}$

$$= \frac{F/A}{Y} = \frac{F}{AY}$$

$$= \frac{122500}{(3.14 \times 0.27) \times 2 \times 10^{11}}$$

$$= 0.722 \times 10^{-6} = 7.22 \times 10^{-7}$$

Address: Plot No 420, Behind Shopprix Mall, Vaishali Sector 5, Ghaziabad – 201010
M: 9999907099, 9818932244 O: 0120-4130999
Website: www.vaishalieducationpoint.com , www.educationsolution.co