MATHEMATICS CLASS XII
CHAPTER -6 APPLICATIONS OF DERIVATIVES

Q.1. Find the rate of change of the area of a circle with respect to its radius r when r = 3 cm.

Q.2. A balloon which always remains spherical, has a variable diameter $\frac{3}{2}(2x + 3)$. Determine the rate of change of volume with respect to x.

Q.3. Find the rate of change of the curved surface of a right circular cone of radius r and height h with respect to change in radius.

Q.4. A stone is dropped into a quiet lake and waves move in circles at a speed of 3.5 cm per second. At the instant when the radius of the circular wave in 7.5 cm, how fast is the enclosed area increasing ?

Q.5. If the area of a circle increases at a uniform rate, then prove that the perimeter varies inversely as the radius.

Q.6. A balloon which always remains spherical is being inflated by pumping in 900 cubic centimeters of gas per second . Find the rate at which the radius of the balloon is increasing when its radius is 15 cm.

Q.7. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3cm/sec. How fast is the area decreasing when two equal sides are equal to the base.

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Q.8. The volume of a spherical balloon is increasing at the rate of 25cm^3/ sec. Find the rate of change of its surface area when its radius is 5 cm.

Q.9. From a cylindrical drum containing petrol and kept vertical, the petrol is leaking at the rate of 10 cm^3/sec. If the radius of the drum is 25 cm and height 1 metre, find the rate at which the level of the petrol is changing when the petrol level is 80 cm.

Q.10. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which y – coordinate is changing 8 times as fast as the x – coordinate.

Q.11. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2m/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

Q.12. A man of height 2 metres walks at a uniform speed of 5 kilometer/ hour away from a lamp post which is 6 metres high. Find the rate at which the length of his shadow increases.

Q.13. A man is moving away from a tower 49.6 m high at the rate of 2m/sec. Find the rate at which angle of elevation of the top of the tower is changing, when he is at a distance of 36 m from the foot of the tower. Assume that the eye level of the man is 1.6 m from the ground.

Q.14. Two men A and B start with velocities v at the same time from the junction of two roads inclined at 45° to each other. If they travel by different roads, find the rate at which they are being separated.

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Q.15. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi – vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10 m.

Q.16. The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (marginal revenue). If the total revenue (in rupees) received from the sale of x-units of a product is given by $R(x) = 3x^2 + 36x + 5$.

Q.17. The amount of pollution content added in air in a city due to x – diesel vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$.

Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question.

Q.18. If the sides of a square are decreasing at the rate of 1.5 cm/sec, at what rate is its area increasing when its side is 10 cm ?

Q.19. Find the rate of change of the surface area of a sphere of radius r with respect to change in the radius when radius is 2 m.

Q.20. Find the rate of change of the whole surface of a closed circular cylinder of radius r and height h with respect to change in radius.

Q.21. Sand is pouring from a pipe at the rate of $12 cm^3$/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always

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one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm ?

Q.22. Water is dripping out at a steady rate of 1 cu cm/ sec through a tiny hole at the vertex of the conical vessel, whose axis is vertical. When the slant height of water in the vessel is 4 cm, find the rate of decrease of slant height, where the semi-vertical angle of the cone is $\frac{\pi}{6}$.

Q.23. $p(x) = 0.03\,x^3 + 0.2\,x^2 + 15x + 100$ represents the air pollution in an industrial area due to smoke produced by x chimneys. Find the marginal value of air pollution when 3 chimney are increased. Which value does this question indicate ?

Q.24. The curve $3y^2 = 2ax^2 + 6b$ passes through the point P(3,-1) and the gradient of the curve at P is -1. Find the values of a and b.

Q.25. If the tangent to the curve $y = x^3 + ax + b$ at P(1 , - 6) is parallel to the line $y - x = 5$, find the values of a and b.

Q.26. At what point on the circle $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangents are parallel to x – axis?

Q.27. At what points on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangents are parallel to y – axis?

Q.28. Find the points on the curve $9y^2 = x^3$ where the normal to the curve makes equal intercepts on the coordinate axes.

Q.29. Find the equation of the normal to the curve $x^2 + 2y^2 - 4y - 6y + 8 = 0$ at the point whose abscissa is 2.

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Q.30. Show that the equation of normal at any point on the curve $x = 3\cos\theta - \cos^3\theta$, $y = 3\sin\theta - \sin^3\theta$ is $4(y\cos^3\theta - x\sin^3\theta) = 3\sin 4\theta$.

Q.31. Find the equations of tangents to the curve $3x^2 - y^2 = 8$ which passes through the point $\left(\frac{4}{3}, 0\right)$.

Q.32. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1,2). Also find the equation of the corresponding tangent.

Q.33. Find the equations of tangents to the curve $y = \frac{1}{x-1}$, which have slope -1.

Q.34. Find the equation of the tangent to the curve $x^2 + 3y = 3$, which is parallel to the line $y - 4x + 5 = 0$.

Q.35. Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$.

Q.36. Find the equations of tangent lines to the curve $y = 4x^3 - 3x + 5$ which are perpendicular to the line $9y + x + 3 = 0$.

Q.37. Find the equations of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

Q.38. Prove that the curves $y^2 = 4x$ and $x^2 + y^2 - 6x + 1 = 0$ touch each other at the point (1,2).

Q.39. Find the angle of intersection between the following curves :

(i) $xy = 3$ and $xy^2 = 6$ (ii) $y^2 = 4x$ and $x^2 = 4y$.

Q.40. Show that the curves $x = y^2$ and $xy = k$ cut orthogonally if $8k^2 = 1$.

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Q.41. Find the point on the curve $y = 3x^2 - 2x + 1$ at which the slope of the tangent is 4.

Q.42. Determine the point on the curve $y = 3x^2 - 1$ at which the slope of the tangent is 3.

Q.43. Find the point on the curve $y = 3x^2 - 12x + 5$ at which the tangent is parallel to x – axis.

Q.44. At what point on the curve $y = x^2$ does the tangent make an angle of 45° with the x-axis?

Q.45. If the slope of the tangent to the curve $y = x^3 + ax + b$ at the point (1, - 6) is -1, find the values of a and b.

Q.46. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}$ at x = 10.

Q.47. If the tangent to the curve $y = x^3 + ax + b$ at (1, - 6) is parallel to the line 2x – 2y + 7 = 0, find a and b.

Q.48. Find a point on the curve $y = x^2$ where the slope of the tangent is equal to the x – coordinate of the point.

Q.49. Find a point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the chord joining the points (2,0) and (4,4).

Q.50. Determine the point of the curve $y =(x - 2)^2$ at which the tangent is parallel to the chord joining the points (2,0) and (4,4).

Q.51. Find the points on the curve $4y = x^3$ where the slope of the tangent is $\frac{16}{3}$.

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Q.52. Find the points on the curve $x^2 + y^2 = 13$, the tangent at each one of which is parallel to the line 2x + 3y = 7.

Q.53. Find all points on the curve $y = 4x^3 – 2x^5$ at which the tangents pass through origin.

Q.54. Given $y = x^4 – 10$ and x changes from 2 to 1.99, what is approximate change in y.

Q.55. Find the approximate value of (2.01), where $f(x) = 4x^2 + 5x + 2$.

Q.56. A circular metal plate expands under heating so that its radius increases by 2%. Find the approximate increase in the area of the plate before heating is 10 cm.

Q.57. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area.

Q.58. Find the approximate value of tan 46°, given that 1° = 0,01745 radians.

Q.59. Find the percentage error in computing the surface area of a cuboidal box if an error of 1% is made in measuring the lengths of the edges of the box.

Q.60. Solve : x(x – 1) (x – 2) (x – 3) > 0

Q.61. Determine the values of x for which $\frac{x^2-2x+5}{3x^2-2x-5} \geq \frac{1}{2}$.

Q.62. Solve for x :

(i) x(x – 2) (x – 5) (x + 5) > 0 (ii) $x^4 – 5x^2 + 4 \geq 0$.

Q.63. Find all real values of x which satisfy

(i) $x^3 (x – 1)(x – 2) > 0$ (ii) $x^2 (x – 1) (x – 2) \leq 0$.

Q.64. Solve for x :

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(i) $\frac{1}{x-2} \leq 1$ (ii) $\frac{x^2-3x+24}{x^2-3x+3} < 4$.

Q.65. Show that the function f(x) = 1 - $\frac{1}{x}$ is strictly increasing.

Q.66. Prove that the function f(x) = 3 + $\frac{1}{x}$ is strictly decreasing .

Q.67. For what value of k, the function f(x) = k(x + sin x) + k is increasing ?

Q.68. For what value of k, the function f(x) = $kx^3 + 5$ is decreasing ?

Q.69. Find the interval in which the function f(x) = x^2e^{-x} is increasing.

Q.70. Prove that the function f(x) = ax + b is strictly decreasing iff a <0.

Q.71. Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$?

(i) cos x (ii) cos 2x (iii) $\sin^2 x$ (iv) tan x.

Q.72. Prove that the function f(x) = $x^3 - 3x^2 + 3x - 100$ is strictly increasing on R.

Q.73. Prove that the function f(x) = sin x is

(i) strictly increasing in $\left(0, \frac{\pi}{2}\right)$ (ii) strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

Q.74. Prove that the function f(x) = $|x|$ is

(i) strictly increasing in (0,∞) (ii) strictly decreasing in (-∞ , 0).

Q.75. Find the intervals in which the fuction f(x) = 2 log (x – 2) – x^2 + 4x – 5 is strictly increasing or strictly decreasing.

Q.76. Find the intervals in which the function given by f(x) = x^2e^x is strictly increasing or strictly decreasing.

Q.77. Prove that the function f(x) = sin $\left(2x + \frac{\pi}{4}\right)$ is decreasing for $\frac{3\pi}{8} \leq x \leq \frac{5\pi}{8}$.

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Q.78. Find the maximum and minimum values of the functions f given by f(x) = sin x + cos x.

Q.79. Find the maximum and minimum values, if any, of the following fuctions :

(i) x + 1 in (-1, 1) (ii) x + 1 in [-1 , 1].

Q.80. Find the absolute maximum and minimum values of :

(i) $f(x) = 2x^3 - 9x^2 + 12x - 5$ in [0,3]

(ii) $f(x) = x^2\sqrt{1+x}$ in $\left[-1, \frac{1}{2}\right]$

(iii) $f(x) = 12x^{4/3} - 6x^{1/3}$, $x \in [-1,1]$.

Q.81. Find the maximum and the minimum values of :

(i) $f(x) = \sin x + \frac{1}{2}\cos 2x$ in $0 \le x \le \frac{\pi}{2}$.

(ii) $f(x) = 3 + |x-2|$ in $-2 \le x \le 5$.

Also find points of maxima and minima.

Q.82. Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Q.83. It is given that at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value in the interval [0,2]. Find the value of a.

Q.84. What are the minimum and maximum values of 2 – 3 cos x ?

Q.85. What are the maximum and minimum values of 3 sin x + 4 cos x ?

Q.86. Determine the maximum value of f(x) = sin 2x in $\left[0, \frac{\pi}{2}\right]$.

Q.87. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

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Q.88. Find the stationary points of the function $f(x) = 3x^4 - 8x^3 + 6x^2$ and distinguish between them. Also find the local maximum and local minimum values, if they exist.

Q.89. Find the points of local maxima and minima of the function $f(x) = 3x^4 - 4x^3 + 5$ in [-1 , 2]. Also find absolute maximum and minimum values.

Q.90. Find all points of local maxima and minima and the corresponding maximum and minimum values of the function f where

$$f(x) = \sin^4 x + \cos^4 x,\ 0 < x < \pi.$$

Q.91. Find the turning points of the following functions and distinguish between them. Also find the local maximum and minimum values of the functions :

(i) $f(x) = 2x^3 - 21x^3 + 36x - 20$ (ii) $f(x) = 4x^3 + 19x^2 - 14x + 3$

(iii) $f(x) = x^3 - 3x^2 + 3x$.

Q.92. Find the coordinates of stationary points on the curve

$$Y = x^3 - 3x^2 - 9x + 7,$$

And distinguish between points of local maxima and minima.

Q.93. Find the maximum value of $\frac{\log x}{x}$, $x > 0$.

Q.94. For what values of a and b, the functions $f(x) = x^3 + ax^2 + bx - 3$ has local maximum value at $x = 0$ and local minimum value at $x = 1$?

Q.95. The sum of three positive numbers is 26. The second is thrice as large as the first. If the sum of the square of these numbers is least, find the numbers.

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Q.96. Let AB and CD be two vertical poles at the points B and D. If AB = 16m, CD = 22 m and BD = 20 m, then find the distance of a point P on BD from the point B such that $AP^2 + CP^2$ is minimum.

Q.97. Prove that the area of a right – angled triangle of given hypotenuse is maximum when the triangle is isosceles.

Q.98. Find the largest possible area of a right – angled triangle whose hypotenuse is 5 cm long.

Q.99. If the sum of the lengths of the hypotenuse and a side of a right-angled triangle is given show that the area of the triangle is maximum when the angle between is $\frac{\pi}{3}$.

Q.100. ABC is a right – angled triangle of given area k. Find the sides of the triangle for which the area of the circumscribed circle is least.

Q.101. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum? Also find its maximum volume.

Q.102. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given right circular cone is half that of the cone.

Q.101. Show that the volume of the largest cone that can be inscribed in a sphere of radius r is $\frac{8}{27}$ of the volume of the sphere.

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Q.102. Show that the altitude of a right circular cone of maximum curved surface which can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

Q.103. A given quantity of metal is to be cast into a half circular cylinder (i.e. with rectangular base and semicircular ends). Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its circular ends is $\pi : (\pi + 2)$.

Q.104. Find the equation of the line through the point (3,4) which cuts from the first quadrant a triangle of minimum area.

Q.105. Three numbers are given whose sum is 180 and the ratio of first two of them is 1 : 2. If the product of the numbers is greatest, find the numbers.

Q.106. Prove that the perimeter of a right – angled triangle of given hypotenuse is maximum when the triangle is isosceles.

Q.107. Two sides of a triangle have lengths a and b, and the angle between them is θ. What value of θ will maximize the area of the triangle? Also find the maximum area of the triangle.

Q.108. A sheet of paper is to contain 18 cm^2 of printed matter. The margins at the top and bottom are 2 cm each, and at the sides 1 cm. Find the dimensions of the sheet which require the least amount of paper.

Q.109. A window is in the form of a rectangle surmounted by a semicircle opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

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Q.110. A window is in the form of a rectangle above which there is a semicircle. If the perimeter of the window is p cm, show that the window will admit maximum possible light only when the radius of semicircle is $\frac{p}{\pi+4}$ cm.

Q.111. Show that the surface area of a closed cuboid with square base and given volume is minimum when it is a cube.

Q.112. An open box with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width.

Q.113. Show that the maximum volume of the cylinder which can be inscribed in a sphere of radius $5\sqrt{3}$ cm is 500π cm^3.

Q.114. A manufacturer plans to construct a cylindrical can to hold one cubic metre of oil. If the cost of constructing top and bottom of the can is twice the cost of constructing the side, what are the dimensions of the most economical can ?

Q.115. Find the equations of tangents to the curve y = cos (x + y), $-2\pi \leq x \leq 2\pi$, that are parallel to the line x + 2y = 0.

Q.116. The curve $y = ax^3 + bx^2 + cx + 5$ touches the x- axis at P(-2,0) and cuts y – axis at a point Q where its gradients is 3. Find a, b, c.

Q.117. Show that the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ will cut orthogonally if $\frac{1}{a} - \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}$.

Q.118. Prove that the functions f(x) = [x] – x is decreasing on [0,1).

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Q.119. Use the function $x^{1/x}$, x > 0, to determine the bigger of e^{π} and π^{e}.

Q.120. If y = a log $|x|$ + b x^2 + x has extreme values at x = -1 and at x = 2, then find a and b.

Q.121. If y = $\frac{ax-b}{(x-1)(x-4)}$ has a turning point P(2, -1), find the values of a and b and show that y is maximum at P.

Q.122. If f(x) = $\sin^3 x + k \sin^2 x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, then find the interval in which k should lie in order that f(x) has exactly one maximum value and exactly one minimum value.

Q.123. A swimmer P is in a sea at a distance d km from the nearest point A on a straight shore. The house of the swimmer is on the shore at a distance c km from A. The ratio of his rate of walking to the rate of swimming is sec α. At what point on the shore should he land so that he reaches his house in shortest possible time ?

Q.124. A box of constant volume c is to be twice as long as it is wide. The cost per unit area of the material on the top and four side faces is three times the cost for bottom. What are the most economical dimensions of the box?

Q.125. A window of fixed perimeter (including the base of the arch) is in the form of a rectangle surmounted by a semicircle. The circular portion is fitted with colour glass while the rectangular part is fitted with clear glass. The clear glass transmits three times as much light per square unit as the coloured glass

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does. What is the ratio of the sides of the rectangle so that the window transmits maximum light?

Q.126. Find the maximum surface area of a circular cylinder than can be inscribed in a sphere of radius R.

Q.127. Find the dimensions of the right circular cone of minimum volume that can be circumscribed about a sphere of radius 8 cm.

Q.128. Find the co-ordinates of the point on the parabolas $y^2 = x$ whose distances from the line $y - x = 1$ is shortest. Also find the shortest distance.

Q.129. The semi-vertical angle of a right circular cone is 45°. Find approximately the lateral surface area of the cone when its height changes from 20 cm to 20.025 cm.

Q.130. A wire of length 25 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces so that the combined area of the square and the circle is minimum.

Q.131. Find the dimensions of the rectangle of maximum area that can be inscribed in the portion of the parabola $y^2 = 4px$ intercepted by the line $x = a$.

Q.132. Show that the maximum volume of a cylinder which can be inscribed in a cone of height h and semi-vertical angle 30° is $\frac{4}{81}\pi h^3$.

Q.133. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is $8m^3$. If building of tank

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costs ₹ 70 per sq. metre for the base and ₹ 45 per m^2 for sides, what is the cost of least expensive tank?

Q.134. Find the semi-vertical angle of the cone of maximum curved surface that can be inscribed in a sphere of radius R.

Q.135. Show that the normal at any point 'θ' to the curve

$x = a\sin^3 t$, $y = b\cos^3 t$ at any point 't'.

Q.136. Find the approximate change in the volume of a cube of side x metres caused by increasing the side by 3%.

Q.137. If $f(x) = 3x^2 + 15x + 5$, then find the approximate value of f(3.02).

Q.138. Determine for which values of x, the function $f(x) = \frac{x}{x^2+1}$ is increasing and for which values of x, it is decreasing . Find also the points on the graph of the function at which the tangent is parallel to x-axis.

Q.139. Find the intervals in which the functions f given by

$$f(x) = \frac{4\sin x - 2x - x\cos x}{2+\cos x},\ 0 \le x \le 2\pi, \text{ is}$$

(i) Increasing (ii) decreasing.

Q.140. Divide the number 4 into two positive numbers such that the sum of square of one and the cube of other is minimum.

Q.141. Find the point on the curve $y^2 = 2x$ which is nearest to the point A(1, -4).

Q.142. A wire of length 25 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of

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the two pieces so that the combined area of the square and the circle is minimum.

Q.143. Find the dimensions of the rectangle of maximum area that can be inscribed in the portion of the parabola $y^2 = 4px$ intercept by the line x = a.

Q.144. If y = $\frac{ax-b}{(x-1)(x-4)}$ has a turning point at P(2,-1), find the values of a and b and show that y is maximum at p.

Q.145. Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$.

Q.146. Prove that tan x > x for all x $\in \left(0, \frac{\pi}{2}\right)$.

Q.147. Prove that the semi – vertical angle of a right circular cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

Q.148. Show that the surface area of a closed cuboid with square base and given volume is minimum when it is a cube.

Q.149. Of all closed cylinder can (right circular), of a given volume, which is open at the top, has minimum total surface area if its height is equal to radius of its base.

Q.150. Find the volume of the largest cylinder that can be inscribed in a sphere of radius r.

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