



## MATHEMATICS CLASS XII

### CHAPTER – 2 INVERSE TRIGONOMETRIC

### FUNCTIONS

Q.1. Find the principal values of :

(i)  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$       (ii)  $\operatorname{cosec}^{-1}(2)$       (iii)  $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ .

Q.2. Let  $\operatorname{cosec}^{-1}(2) = x$ ,  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ ,  $x \neq 0$

Q.3. Using principal values, find the values of :

(i)  $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$       (ii)  $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ .

Q.4. Evaluate :  $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ .

Q.5. If  $\tan^{-1}\left(\frac{4}{3}\right) = x$ , find the value of  $\cos x$ .

Q.6. Evaluate the following:

(i)  $\sin\left(2 \cos^{-1}\left(-\frac{3}{5}\right)\right)$       (ii)  $\sin\left(2 \cos^{-1}\left(-\frac{5}{12}\right)\right)$

(iii)  $\tan\left(\frac{1}{2} \cos^{-1}\frac{\sqrt{5}}{3}\right)$ .

Q.7. Show that :

(i)  $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) \neq \frac{5\pi}{6}$ , what is its value?

(ii)  $\cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right) \neq \frac{\pi}{6}$ , what is its value?

(iii)  $\sin^{-1}\left(\sin\frac{5\pi}{3}\right) \neq \frac{5\pi}{3}$ . what is its value?



Q.8. Using principal values, evaluate the following:

$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right).$$

Q.9. Find the value of  $\sec(\tan^{-1}(-\sqrt{3}))$ .

Q.10. Find the domain of the following functions :

(i)  $\sin x + \sin^{-1} x$       (ii)  $\sin^{-1}(1 - x)$ .

Q.11. Find the domain of the function  $\cos^{-1}(3x - 2)$ .

Q.12. Find the domain of the function  $\sec^{-1}(2x - 3)$ .

Q.13. Write the range of one branch of  $\sin^{-1} x$ , other than the principal branch.

Q.14. Write the range of one branch of  $\cos^{-1} x$ , other than the principal branch.

Q.15. If  $\cot^{-1}\left(\frac{3}{4}\right) = x$ , find the value of  $\cos x$ .

Q.16. (i) If  $\tan^{-1}\left(\frac{3}{4}\right) = x$ , find the values of  $\cos x$  and  $\sin x$ .

(ii) If  $\cot^{-1}\left(-\frac{1}{7}\right) = x$ , find the values of  $\sin x$  and  $\cos x$ .

Q.17. Evaluate the following :

(i)  $\cot(\tan^{-1}\sqrt{3})$       (ii)  $\sin\left(\frac{\pi}{6} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

(iii)  $\cos\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right)$

Q.18. Prove the following :

(i)  $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{2\pi}{3}$       (ii)  $\tan^{-1}(-1) + \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{\pi}{2}$

Q.19. Using principal values, find the values of :

(i)  $\cos^{-1}\left(-\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$       (ii)  $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$



(iii)  $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$       (iv)  $\operatorname{cosec}^{-1}(-1) + \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ .

Q.20. Find the values of the following :

(i)  $\tan^{-1}\left(2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right)$       (ii)  $\tan^{-1}\left(2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right)$

Q.21. Find the value of :

(i)  $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

Q.22. Evaluate the following :

(i)  $\cos\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)$       (ii)  $\operatorname{cosec}\left(\cos^{-1}\left(-\frac{12}{13}\right)\right)$       (iii)  $\sin\left(2\sin^{-1}\frac{3}{5}\right)$ .

Q.23. (i)  $\operatorname{cosec}^{-1}x = \sin^{-1}\left(\frac{1}{x}\right), |x| \geq 1$       (ii)  $\sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right), |x| \geq 1$

(iii)  $\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right), x > 0$ .

Q.24. Find the value of  $\cos(\sec^{-1}x + \operatorname{cosec}^{-1}x), |x| \geq 1$ .

Q.25. Show that :

(i)  $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x, |x| \leq \frac{1}{\sqrt{2}}$

(ii)  $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x, \frac{1}{\sqrt{2}} \leq x \leq 1$ .

Q.26. Write the following functions in the simplest form :

(i)  $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$       (ii)  $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$

Q.27. If  $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$ , prove that  $\sin y = \tan^2\frac{x}{2}$ .

Q.30. Prove that  $\cos^{-1}\left(\frac{\cos\alpha + \cos\beta}{1 + \cos\alpha \cos\beta}\right) = 2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$ .

Q.31. Prove the following :



$$(i) \tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \left( \frac{1-x}{1+x} \right)$$

$$(ii) \frac{1}{2} \tan^{-1} x = \cos^{-1} \left( \sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}} \right).$$

Q.32. Prove that  $\cos (\tan^{-1} (\sin (\cot^{-1} x))) = \sqrt{\frac{1+x^2}{2+x^2}}$ .

Q.33. Solve :

$$(i) \cos^{-1} (\sin(\cos^{-1} x)) = \frac{\pi}{3}$$

$$(ii) 2 \sin^{-1} \left( \frac{2x}{1+x^2} \right) - 4 \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right) + 2 \tan^{-1} \left( \frac{2x}{1-x^2} \right) = \frac{\pi}{3}$$

Q.34. Prove that  $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$ .

Q.35. Prove that  $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$ .

Q.36. Find the value of :  $\tan^{-1} \left( \frac{x}{y} \right) - \tan^{-1} \left( \frac{x-y}{x+y} \right)$ .

Q.37. Prove that :  $4 (\cot^{-1} 3 + \operatorname{cosec}^{-1} \sqrt{5}) = \pi$ .

Q.38. Solve the following equations for x :

$$(i) \sin^{-1} (1-x) - 2 \sin^{-1} x = \frac{\pi}{2}$$

$$(ii) \sin^{-1} x + \sin^{-1} (1-x) = \cos^{-1} x$$

$$(iii) \sin^{-1} \left( \frac{5}{x} \right) + \sin^{-1} \left( \frac{12}{x} \right) = \frac{\pi}{2}.$$

Q.39. Prove that :  $\sin (\cot^{-1} (\cos(\tan^{-1} x))) = \sqrt{\frac{x^2+1}{x^2+2}}$ .

Q.40. Prove that :  $\tan^{-1} \left( \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \right) = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$ .



Q.41. Find the value of  $\tan \frac{1}{2} \left( \sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-y^2}{1+y^2} \right)$ ,  $|x| < 1$ ,  $y > 0$ ,  $xy < 1$ .

Q.42. Prove that  $\sec^2 (\tan^{-1} 2) + \operatorname{cosec}^2 (\cot^{-1} 3) = 15$ .

Q.43. If  $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi m$  prove that  $x + y + z = xyz$ .

Q.44. If  $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$ , prove that  $xy + yz + zx = 1$ .

Q.45. If the angle C of triangle is right angle, prove that

$\tan^{-1} \frac{a}{b+c} + \tan^{-1} \frac{b}{c+a} = \frac{\pi}{4}$ , where a, b, c are the lengths of the sides of the triangle opposite to the angles A, B, C respectively.

Q.46. Find the values of :

(i)  $\sin (\sin^{-1} x + \cos^{-1} x)$ ,  $|x| \leq 1$  (ii)  $\cot (\tan^{-1} x + \cot^{-1} x)$ ,  $x \in \mathbb{R}$

Q.47. Find the values of :

(i)  $\tan^{-1} \left( \sin \left( -\frac{\pi}{2} \right) \right)$  (ii)  $\cos \left( \tan^{-1} \frac{3}{4} \right)$

(iii)  $\tan \left( \cos^{-1} \frac{8}{17} \right)$ .

Q.48. (i) If  $\tan^{-1} x = \frac{\pi}{10}$  for some  $x \in \mathbb{R}$ , find the value of  $\cot^{-1} x$ .

(ii) If  $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$ , then find the value of  $\cos^{-1} x + \cos^{-1} y$ .

Q.49. If  $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$ , then prove that

$$x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2).$$

Q.50. Solve :  $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(3x - 2)$ .

Q.51. Solve the following simultaneously equations :

$$\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}, \quad \cos^{-1} x - \cos^{-1} y = \frac{\pi}{3}.$$



Q.52. Solve :  $\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\left(\frac{23}{36}\right)$ .

Q.53. If  $x = \tan^{-1}(2 \tan^2 x) - \frac{1}{2} \sin^{-1}\left(\frac{3 \sin 2x}{5+4 \cos 2x}\right)$ , find the general values of  $x$ .

Q.54. Solve the following simultaneously equations :

$$\sin^{-1} x - \sin^{-1} y = \frac{\pi}{3}, \cos^{-1} x + \cos^{-1} y = \frac{2\pi}{3}.$$

Q.55. Find all positive integral solutions of

$$\tan^{-1} x + \cos^{-1} \frac{y}{\sqrt{1+y^2}} = \sin^{-1} \frac{3}{\sqrt{10}}.$$

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