

# VAISHALI EDUCATION POINT

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Class – XII

Subject – Mathematics

1	<input type="checkbox"/>	Prove that the function $f(x) = 5x - 3$ is continuous at $x = 0$ , at $x = -3$ and at $x = 5$ . <span style="float: right;">1</span>
2	<input type="checkbox"/>	Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$ . <span style="float: right;">1</span>
3	<input type="checkbox"/>	Examine the following functions for continuity. (a) $f(x) = x - 5$ (b) $f(x) = \frac{1}{x - 5}, x \neq 5$ (c) $f(x) = \frac{x^2 - 25}{x + 5}, x \neq -5$ (d) $f(x) =  x - 5 $ <span style="float: right;">1</span>
4	<input type="checkbox"/>	Prove that the function $f(x) = x^n$ is continuous at $x = n$ , where $n$ is a positive integer. <span style="float: right;">1</span>
5	<input type="checkbox"/>	Is the function $f$ defined by $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$ continuous at $x = 0$ ? At $x = 1$ ? At $x = 2$ ? <span style="float: right;">1</span>
6	<input type="checkbox"/>	Find all points of discontinuity of $f$ , where $f$ is defined by $f(x) = \begin{cases} 2x + 3, & \text{if } x \leq 2 \\ 2x - 3, & \text{if } x > 2 \end{cases}$ <span style="float: right;">1</span>
7	<input type="checkbox"/>	Find all points of discontinuity of $f$ , where $f$ is defined by $f(x) = \begin{cases}  x  + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$ <span style="float: right;">1</span>
8	<input type="checkbox"/>	Find all points of discontinuity of $f$ , where $f$ is defined by $f(x) = \begin{cases} \frac{ x }{x} & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ <span style="float: right;">1</span>
9	<input type="checkbox"/>	Find all points of discontinuity of $f$ , where $f$ is defined by $f(x) = \begin{cases} \frac{x}{ x }, & \text{if } x < 0 \\ -1, & \text{if } x \geq 0 \end{cases}$ <span style="float: right;">1</span>
10	<input type="checkbox"/>	Find all points of discontinuity of $f$ , where $f$ is defined by $f(x) = \begin{cases} x + 1, & \text{if } x \geq 1 \\ x^2 + 1, & \text{if } x < 1 \end{cases}$ <span style="float: right;">1</span>
11	<input type="checkbox"/>	Find all points of discontinuity of $f$ , where $f$ is defined by $f(x) = \begin{cases} x^3 - 3, & \text{if } x \leq 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases}$ <span style="float: right;">1</span>
12	<input type="checkbox"/>	Find all points of discontinuity of $f$ , where $f$ is defined by

	$f(x) = \begin{cases} x^{10} - 1, & \text{if } x \leq 1 \\ x^2, & \text{if } x > 1 \end{cases}$	1
13	<p>Is the function defined by</p> $f(x) = \begin{cases} x + 5, & \text{if } x \leq 1 \\ x - 5, & \text{if } x > 1 \end{cases}$ <p>a continuous function?</p>	1
14	<p>Discuss the continuity of the function f, where f is defined by</p> $f(x) = \begin{cases} 3, & \text{if } 0 \leq x \leq 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \leq x \leq 10 \end{cases}$	4
15	<p>Discuss the continuity of the function f, where f is defined by</p> $f(x) = \begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \leq x \leq 1 \\ 4x, & \text{if } x > 1 \end{cases}$	4
16	<p>Discuss the continuity of the function f, where f is defined by</p> $f(x) = \begin{cases} -2, & \text{if } x \leq -1 \\ 2x, & \text{if } -1 < x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$	4
17	<p>Find the relationship between a and b so that the function f defined by</p> $f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$ <p>is continuous at <math>x = 3</math>.</p>	6
18	<p>For what value of <math>\lambda</math> is the function defined by</p> $f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$ <p>continuous at <math>x = 0</math>? What about continuity at <math>x = 1</math>?</p>	6
19	<p>Show that the function defined by <math>g(x) = x - [x]</math> is discontinuous at all integral point. Here <math>[x]</math> denotes the greatest integer less than or equal to <math>x</math>.</p>	6
20	<p>Is the function defined by <math>f(x) = x^2 - \sin x + 5</math> continuous at <math>x = p</math>?</p>	6
21	<p>Discuss the continuity of the following functions.</p> <p>(a) <math>f(x) = \sin x + \cos x</math>  (b) <math>f(x) = \sin x - \cos x</math>  (c) <math>f(x) = \sin x \times \cos x</math></p>	6
22	<p>Discuss the continuity of the cosine, cosecant, secant and cotangent functions,</p>	6
23	<p>Find the points of discontinuity of f, where</p> $f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ x + 1, & \text{if } x \geq 0 \end{cases}$	6
24	<p>Determine if f defined by</p> $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ <p>is a continuous function?</p>	6

25	<p>Examine the continuity of f, where f is defined by</p> $f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1 & \text{if } x = 0 \end{cases}$ <p style="text-align: right;">6</p>
26	<p>Find the value of K if the function <math>f(x) = \begin{cases} kx^2, &amp; x \geq 1 \\ 4, &amp; x &lt; 1 \end{cases}</math> is continuous at <math>x = 1</math>.</p> <p style="text-align: right;">(2007) 1</p>
27	<p>Discuss the continuity of the function <math>f(x) = \begin{cases} x+1 &amp; \text{if } x \geq 1 \\ x^2+1 &amp; \text{if } x &lt; 1 \end{cases}</math> at <math>x = 1</math></p> <p style="text-align: right;">(2007) 4</p>
28	<p>Differentiate <math>\sin(x^2 + 1)</math> with respect to x from first principle. (2007) 4</p>
29	<p>If <math>y = \sin(\log x)</math>, prove that <math>x^2 \cdot \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0</math>.</p> <p>OR</p> <p>Verify Rolle's theorem for the function : <math>f(x) = x^2 - 5x + 4</math> on <math>[1, 4]</math> (2007) 4</p>
30	<p>Differentiate <math>\sin\sqrt{2x}</math> with respect to x from first principle. (2007) 4</p>
31	<p>If the following function <math>f(x)</math> is continuous at <math>x = 0</math>, find the value of k:</p> $f(x) = \begin{cases} \frac{1 - \cos 2x}{2x^2} & x \neq 0 \\ k & x = 0 \end{cases}$ <p style="text-align: right;">(2008) 4</p>
32	<p>If <math>y = (\log x)^{\cos x} + \frac{x^2 + 1}{x^2 - 1}</math>,</p> <p>OR</p> <p>If <math>x = a \left( \cos t + \log \tan \frac{t}{2} \right)</math> and <math>y = a \sin t</math>, find <math>\frac{dy}{dx}</math> (2008) 4</p>
33	<p>For what value of k is the following function continuous at <math>x = 2</math>?</p> $f(x) = \begin{cases} 2x + 1 & ; x < 2 \\ k & ; x = 2 \\ 3x - 1 & ; x > 2 \end{cases}$ <p style="text-align: right;">(2008) 4</p>
34	<p>Differentiate the following with respect to x : <math>\tan^{-1} \left( \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right)</math> (2008) 4</p>
35	<p>If the function defined by <math>f(x) = \begin{cases} 2x - 1 &amp; , x &lt; 2 \\ a &amp; , x = 2 \\ x + 1 &amp; , x &gt; 2 \end{cases}</math> is continuous at <math>x = 2</math>, find the value of a. Also discuss the continuity of <math>f(x)</math> at <math>x = 3</math> (2009 Comp.) 4</p>
36	<p>If <math>y = (\log x)^x + (x)^{\cos x}</math>, find <math>\frac{dy}{dx}</math>.</p> <p>OR</p> <p>If <math>x = a \left( \cos \theta + \log \tan \frac{\theta}{2} \right)</math> and <math>y = a \sin \theta</math>, find the value of <math>\frac{d^2y}{dx^2}</math> at <math>\theta = \frac{\pi}{4}</math> (2009 comp.) 4</p>
37	<p>Differentiate the following function w. r. t. x: <math>x^{\sin x} + (\sin x)^{\cos x}</math> (2009) 4</p>
38	<p>Find <math>\frac{dy}{dx}</math> if <math>(x^2 + y^2)^2 = xy</math></p> <p>OR</p> <p>If <math>y = 3 \cos(\log x) + 4 \sin(\log x)</math>, then show that <math>x^2 \cdot \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0</math> (2009) 4</p>

39	$f(x) = \begin{cases} k(x^2 + 2), & \text{if } x \leq 0 \\ 3x + 1, & \text{if } x > 0 \end{cases}$ <p>For what value of k is the function defined by continuous at <math>x = 0</math>? Also write whether the function is continuous at <math>x = 1</math>. <b>(2010 Comp.) 4</b></p>
40	<p>Show that the function <math>f(x)</math> defined as follows, is continuous at <math>x = 2</math>, but not differentiable there at : <math>x = 2</math></p> $f(x) = \begin{cases} 3x - 2 & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x \leq 2 \\ 5x - 4, & x > 2 \end{cases}$ <p>OR</p> <p>Find <math>\frac{dy}{dx}</math>, if <math>y = \sin^{-1} [x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}]</math> <b>(2010) 4</b></p>
41	<p>Discuss the continuity of the function <math>f(x)</math> at <math>x = \frac{1}{2}</math> when <math>f(x)</math> is defined as follows :</p> $f(x) = \begin{cases} \frac{1}{2} + x, & 0 \leq x < \frac{1}{2} \\ 1, & x = \frac{1}{2} \\ \frac{3}{2} + x, & \frac{1}{2} < x \leq 1 \end{cases}$ <p><b>(2011 Comp.) 4</b></p>
42	<p>If <math>x\sqrt{1+y} + y\sqrt{1+x} = 0, (x \neq y)</math>, then prove that <math>\frac{dy}{dx} = \frac{1}{(1+x)^2}</math></p> <p>OR</p> <p>If <math>x = a \left( \cos t + \log \tan \frac{t}{2} \right)</math> and <math>y = a \sin t</math>, find <math>\frac{dy}{dx}</math>. <b>(2011 Comp.) 4</b></p>
43	<p>Find the value of 'a' for which the function <math>f</math> defined as</p> $f(x) = \begin{cases} a \sin \frac{\pi}{2}(x+1), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ <p>is continuous at <math>x = 0</math>. <b>(2011) 4</b></p>
44	<p>Differentiate <math>x^x \cos x + \frac{x^2 + 1}{x^2 - 1}</math> w.r.t.x</p> <p>OR</p> <p>If <math>x = a(\theta - \sin \theta), y = a(1 + \cos \theta)</math>, find <math>\frac{d^2y}{dx^2}</math> <b>(2011) 4</b></p>
45	<p>Examine the continuity of the function <math>f(x) = x^2 + 5</math> at <math>x = -1</math>. <b>1</b></p>
46	<p>Examine the continuity of the function <math>f(x) = \frac{1}{x+3}, x \in \mathbb{R}</math>. <b>1</b></p>
47	<p>Find the value of m, for which the function</p> $f(x) = \begin{cases} m(x^2 - x) & x > 0 \\ \cos x & x \leq 0 \end{cases}$ <p>is continuous at <math>x = 0</math>. <b>1</b></p>
48	<p>If function <math>f(x) = \frac{2x + 3 \sin x}{3x + 2 \sin x}</math>, for <math>x \neq 0</math> is continuous at <math>x = 0</math>, then find <math>f(0)</math>. <b>1</b></p>
49	<p>Show that the function</p> $f(x) = \begin{cases} x^3 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$ <p>is not continuous at <math>x = 0</math>. <b>1</b></p>
50	<p>Find the point of discontinuity, if any, for the function <math>f(x) = \frac{1}{x-5}</math>. <b>1</b></p>

51	<input type="checkbox"/>	State the points of discontinuity for the function $f(x) = [x]$ in $-3 < x < 3$ . 1
52	<input type="checkbox"/>	Is the function $f(x) = \frac{3x+4\tan x}{5x}$ continuous at $x = 0$ ? Give reasons. 1
53	<input type="checkbox"/>	Discuss the differentiability of the function $f(x) = (x-1)^{2/3}$ at $x = 1$ . 1
54	<input type="checkbox"/>	Find the derivative of, $e^{\sqrt{x}+3}$ , with respect to $x$ . 1
55	<input type="checkbox"/>	Differentiate the following w.r.t. $x$ , $y = 5^{\log(\sin x)}$ . 1
56	<input type="checkbox"/>	Find $\frac{dy}{dx}$ , when $\sqrt{x} + \sqrt{y} = 5$ at $(4, 9)$ . 1
57	<input type="checkbox"/>	If $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$ , find $\frac{dy}{dx}$ . 1
58	<input type="checkbox"/>	Given $f(0) = -2$ , $f'(0) = 3$ . Find $h'(0)$ where $h(x) = x f(x)$ . 1
59	<input type="checkbox"/>	Verify LMV theorem for the function : $y = \sqrt{x-2}$ in $[2, 6]$ . 4
60	<input type="checkbox"/>	Find the derivative of $\sin^{-1}\left(\frac{1-x}{1+x}\right)$ , w.r.t. $\sqrt{x}$ . 4
61	<input type="checkbox"/>	If $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ , show that $\frac{dy}{dx} - \sec x = 0$ . 4
62	<input type="checkbox"/>	If $e^x + e^y = e^{x+y}$ , prove that $\frac{dy}{dx} = -e^{y-x}$ . 4
63	<input type="checkbox"/>	Show that the function 'f' defined by $f(x) =  1-x  +  x $ , where $x$ is any real number, is a continuous function. 4
64	<input type="checkbox"/>	Prove that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right] = \sqrt{a^2-x^2}$ . 4
65	<input type="checkbox"/>	If $y = a \cos(\log x) + b \sin(\log x)$ , prove that $x^2 y_2 + x y_1 + y = 0$ . 4
66	<input type="checkbox"/>	If $x = a(\theta - \sin \theta)$ , $y = a(1 - \cos \theta)$ , find $\frac{d^2 y}{dx^2}$ at $\theta = \frac{\pi}{2}$ . 4
67	<input type="checkbox"/>	Find the derivative of $\log\left(\sqrt{\sin x - \cos x}\right)$ , w.r.t. $\sin x$ . 4
68	<input type="checkbox"/>	If $y = (\cot^{-1} x)^2$ , show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 - 2 = 0$ . 4
69	<input type="checkbox"/>	Find the values of $a$ and $b$ , such that the function defined by $f(x) = \begin{cases} 5, & x \leq 2 \\ ax+b, & 2 < x < 10 \\ 21, & x \geq 10 \end{cases}$ is a continuous function. 4
70	<input type="checkbox"/>	Differentiate w.r.t. $x$ : $\log \sqrt{\frac{1-\sin x}{1+\sin x}}$ . 4
71	<input type="checkbox"/>	If $y = \sin\left\{2 \tan^{-1}\left(\frac{1-x}{1+x}\right)\right\}$ , then prove that $\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$ . 4
72	<input type="checkbox"/>	If $(a+bx) \cdot e^{y/x} = x$ , then prove that $x^3 \frac{d^2 y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^2$ . 4
73	<input type="checkbox"/>	If $y = x^{x^{x^{\dots \infty}}}$ , then prove that $\frac{dy}{dx} = \frac{y^2}{x(1-y \log x)}$ . 4

